



A Comprehensive Mathematical Model for a Photovoltaic (PV) Module's Associated Thermal and Electrical Performance

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Abstract

The sun is an inexhaustible source of clean energy. For a long time, humans perceived the sun as a source of heat and light, without thinking of using it fully for their purposes. The worldwide solar photovoltaic (PV) market has been growing rapidly, with significant investments leading to increased capacity. Mathematical models are used to calculate solar radiation, taking into account various meteorological factors to predict solar energy generation. The construction of P-V curves helps understand the output of PV systems, crucial for optimizing energy conversion efficiency. The photovoltaic effect allows solar cells to convert light into electrical energy, highlighting the importance of understanding the relationship between current and voltage in solar panels. In this work, the mathematical model of PV is presented and the equations that describe the relation between PV and environment are presented. Finally, a complete model is designed in Matlab and the results are presented.

Keywords: Renewable Energy, Photovoltaic, Transient Response, Photocurrent, Series Resistance.

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1. Introduction

The worldwide sun oriented photovoltaic (PV) advertise has been developing since 2005 by a normal of 40% per year.

This is often much more than for any other industry [1]. It is anticipated that within the following 20 a long time, sun powered PV will make more than 2 million occupations, and diminish nursery gas emanations into the environment by 350 million tons of CO₂, which is proportionate to halting 140 coal-fired control plants. The whole sun powered PV capacity by 2030 will surpass 650 GW [2, 3]. For the seventh year in a row, the global solar PV market received more investment than other renewable energy industries. In 2016, investments accounted for 57% of the total volume and amounted to \$113.7 billion. USA. Despite an annual decline in total global investment in solar energy, for example by 34% compared to 2015, the newly installed solar photovoltaic capacity increased by 38% and exceeded 80 GW in 2016. Jalil et al., Abdulmunem, et al., Jalil, el at., and Salman et al. [4-7] invested performance of PV cells. The parameters of the PV model have been modified to account for the effects of different temperatures and solar irradiance. The model is then used to calculate maximum power (MPP) under specified environmental conditions.

2. Mathematical models of PV components

2.1. Solar Radiation Model

The sum of essential sun based vitality that's accessible for transformation of the sun based control plant is decided by the intensity of the whole sun oriented radiation at the put of its establishment and depends on the geographic arranges of the area of the sun oriented control plant, the spatial introduction of the SB, as well as on outside meteorological variables:

discuss temperature, cloudiness level, reflection coefficient of the earth's surface.

Most meteorological components are of a stochastic nature, and their vacillations over time contain components from divisions of a moment to at slightest a few decades, which for all intents and purposes prohibits the plausibility of their solid assurance by hypothetical strategies.

At show, within the plan of FES, probabilistic (factual) strategies for calculating radiation characteristics are overwhelmingly utilized, which utilize uncommon climate reference books or electronic databases compiled based on the comes about of long-term meteorological perceptions as starting information.

The overall sun oriented radiation to an accepting surface self-assertively situated in slant and azimuth is calculated agreeing to the strategy proposed by B.Y.H. Liu, R.C. Jordan [8]:

$$I_{total}^{\beta\gamma} = I_d^{\beta\gamma} + I_s^{\beta\gamma} + I_r^{\beta\gamma} = I_r^s \frac{\cos \theta}{\cos \theta_z} + I_s^s \frac{1 + \cos \beta}{2} + \rho I_d^s \frac{1 - \cos \beta}{2} \quad \dots(1)$$

Where $I_d^{\beta\gamma}$, $I_s^{\beta\gamma}$, $I_r^{\beta\gamma}$ values of the entire, coordinate, scattered from clouds and pressurized canned products, and reflected from the earth's surface onto an slanted surface, separately; I_r^s , I_s^s , I_d^s values of coordinate, scattered and add up to sun oriented radiation occurrence on a level surface; θ the point between the course of the sun based radiation flux to the surface and the ordinary to it; θ_z is the zenith angle of the Sun; ρ - albedo of the earth's surface. The value of the angle θ is determined from the solution of the equation [9]:

$$\cos \theta = (A_1 - A_2) \sin \delta + [A_3 \sin \omega + (A_4 + A_5) \cos \omega] \cos \delta : A_1 = \sin \varphi \cos \beta ; \quad \dots(2)$$

$$A_2 = \cos \varphi \sin \beta \cos \gamma ; A_3 = \sin \beta \sin \gamma$$

$$A_4 = \cos \varphi \cos \beta ; A_5 = \sin \varphi \sin \beta \cos \gamma$$

Where: φ latitude of the area at the point of installation of the receiving platform; δ represents the angle of declination of the Sun; β represents the angle of inclination of the receiving platform to the horizontal plane; γ - azimuthal installation angle receiving platform (when directed to the South $\gamma = 0^\circ$, when deviated to the East γ is considered positive, when deviated to the West - negative).

The zenith angle could be written as follow:

$$\theta_z = \arccos[\sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \omega] \quad \dots(3)$$

The Sun's elevation angle above the horizon h could be written as:

$$h = 90 - \theta_z$$

The azimuth angle of the position of the Sun A_z is the solution of the formula:

$$\cos A_z = \frac{\sin h \sin \varphi - \sin \delta}{\cosh \cos \varphi} \quad \dots(4)$$

$$\omega = 15(t - 12 - T_{eq} - \Delta T_G) + \psi \quad \dots(5)$$

Where t is the current official local time, hour; ΔT_G - local official time and Greenwich Mean Time difference, hour; ψ is the geographic longitude location of the receiving site, deg.

The time correction for the formula of time is determined by the formula:

$$T_{eq} = \frac{1}{60} [9,87 \sin(2B) - 7,53 \cos(B) - 1,5 \sin(B)] \quad \dots(6)$$

Where

$$B = B_0(N - 81); B_0 = \frac{360}{365}$$

N changes in range (for 01.01 - $N=1$, for 02.01 - $N=2$, etc.).

The declination angle [9]:

$$\delta = 23,45 \sin[B_0(N + 284)] \quad \dots(7)$$

The angles of sunrise and sunset of the Sun in solar time are determined from the expressions:

$$\omega_{sr} = -\arccos[-tg \varphi tg \delta]; \quad \dots(8)$$

$$\omega_{ss} = \arccos[-tg \varphi tg \delta];$$

To require under consideration the genuine characteristics of sun oriented radiation arriving at a certain region, the reenactment employs the normal values of the air straightforwordness record K_T for a given month, gotten from the database of the US space agency NASA SSE [10] for a 22-year observation period from 1983 to 2004.

It is expected that the straightforwordness list for the current day by day values of sun based radiation is rise to its normal day by day esteem for a given month:

$$K_T = \frac{I_{tot}^s}{I_0} \quad \dots(9)$$

Where extra-atmospheric radiation to a horizontal surface, which could be written as follow [11]:

$$I_0 = I_{sun}[1 + 0,033 \cos(B_0 N)] \cos \theta_z \quad \dots(10)$$

Where $I_{sun} = 1367$ solar constant.

2.2. Construction of P-V CURVES

A PV network employs one or more PV clusters for a changing over sun based vitality into electrical vitality. The basic unit of a PV cluster may be a PV cell and it works on photovoltaic rule. The photovoltaic impact can be characterized as change of light into electrical vitality by implies of external electrons that break the bond in a semiconductor gadget when the photon vitality is higher than the band hole vitality. The photoelectric impact is at the center of the operation of the sun based cell, the photocell changes over photon vitality into electrical vitality. In arrange to decide the relationship between the yield current and voltage of the sun oriented board; we utilize the identical proportionate circuit (Fig.1).

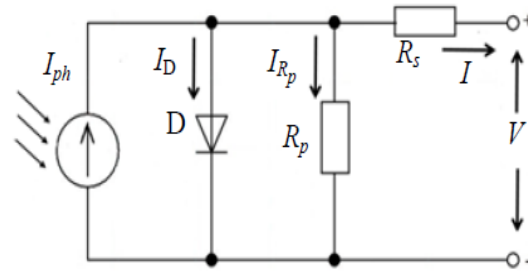


Figure 1. Solar panel equivalent circuit.

The relation between voltage-current of the a PV module is written as:

$$I = I_{pv} - I_{sat} \left[\exp\left(\frac{V + IR_s}{N_s V_T}\right) - 1 \right] - \frac{V + IR_s}{R_p} \quad \dots(11)$$

Where I_{pv} represents photocurrent; I_{sat} the saturation current of diode; R_s is the resistance of equivalent series; R_p is the resistance of equivalent parallel; V is the out voltage; I - out current; V_T represents the voltage of junction thermal, R_s represent series resistance, R_p represent the resistance of parallel and N_s represent the cells number in the PV module which connected in series.

$$V_T = \frac{kTm}{q} \quad \dots(12)$$

Where m represent the factor of ideal diode, k (1.38065×10^{-23} m² kgs⁻²K⁻¹) represent Boltzmann constant, q (1.602176×10^{-19} C) is the electronic charge and T represents the absolute temperature.

Nonlinear Equations could be solved as follow:

$$I = I_{pv} - I_{sat} \exp\left(\frac{V + IR_s}{N_s V_T}\right) - \frac{V + IR_s}{R_p} \quad \dots(13)$$

At the OC point [12]:

$$I_{pv} = I_{sat} \exp\left(\frac{V_{oc}}{N_s V_T}\right) + \frac{V_{oc}}{R_p} \quad \dots(14)$$

Where V_{oc} is the open circuit voltage. Since $\exp\left(\frac{V_{oc}}{N_s V_T}\right)$ is too small it is insignificant, compared to $\exp\left(\frac{V_{oc}}{N_s V_T}\right)$

$$I_{sat} = \left(I_{sc} - \frac{V_{oc} + I_{sc} R_s}{R_p}\right) \exp\left(\frac{-V_{oc}}{N_s V_T}\right) \quad \dots(15)$$

I_{mpp} Could be written as follow in case of MPP:

$$I_{mpp} = I_{pv} - I_{sat} \exp\left(\frac{V_{mpp} + I_{mpp} R_s}{N_s V_T}\right) - \frac{V_{mpp} + I_{mpp} R_s}{R_p} \quad \dots(16)$$

I_{mpp} is the current at the MPP.

$$I_{mpp} = I_{sc} - \frac{V_{mpp} + I_{mpp} R_s - I_{sc} R_s}{R_p} - \left(I_{sc} - \frac{V_{oc} + I_{sc} R_s}{R_p}\right) \exp\left(\frac{V_{mpp} + I_{mpp} R_s - V_{oc}}{N_s V_T}\right) \quad \dots(17)$$

At the MPP with respect to the voltage the derivative of the power is written as [13]:

$$\left.\frac{dI}{dV}\right|_{V=V_{mpp}, I=I_{mpp}} = -\frac{I_{mpp}}{V_{mpp}} \quad \dots(18)$$

So I_{mpp} could be written as:

$$I_{mpp} = \frac{V_{mpp} A \exp(B) + \frac{V_{mpp}}{R_p}}{1 + A \exp(B) R_s + \frac{R_s}{R_p}} \quad \dots(19)$$

Where:

$$A = \frac{I_{sc} R_p - V_{oc} + I_{sc} R_s}{N_s V_T R_p}$$

$$B = \frac{V_{mpp} - V_{oc} + I_{mpp} R_s}{N_s V_T}$$

IV curve at the SC as shown in Fig. 2 could be written as:

$$\left.\frac{dI}{dV}\right|_{V=0, I=I_{sc}} = -\frac{1}{R_{p0}} \quad \dots(20)$$

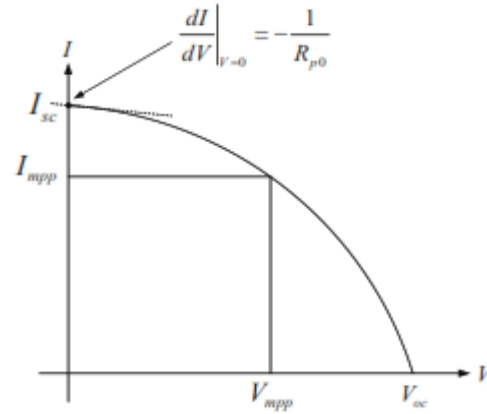


Figure 2. The I-V curve

According to [8], one can obtain:

$$\frac{1}{R_p} = \frac{A \exp(X_1) + \frac{1}{R_p}}{1 + A \exp(X_1) R_s + \frac{R_s}{R_p}} \quad \dots(21)$$

$$X_1 = \frac{I_{sc} R_s - V_{oc}}{N_s V_T} \quad \dots(22)$$

In arrange to watch whether the bends set up by this strategy are reliable with the P-V bends given by the information sheet, we utilize root cruel square mistake (RMSE) given to check the comes about.

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{ic} - x_{id})^2} \quad \dots(23)$$

The effect of Irradiance and temperature environment effect on PV parameter values. For sun powered boards working beneath genuine conditions, the parameters need to be adjusted to the encompassing temperature and irradiance. In this manner, (16) and (17) are connected in case of changing of the temperature or the irradiance.

Where I'_{pv} represents the PV current in case of temperature T and irradiance S , I_{pv} is the PV current in case of standard test condition (STC). STC shows the execution of the sun powered boards and indicate a board temperature of 25 °C and an irradiance of 1000 W/m² with an discuss mass 1.5 range. K_i could be a current steady of the board which is ordinarily recorded within the information sheet, T_n represents the temperature in case of standard test condition and S_n represents the irradiance in case of standard test condition.

$$I'_{sat} = I_{sat} \left(\frac{T}{T_n}\right)^3 \exp\left[\frac{qE_{gn}}{mk} \left(\frac{1}{T_n} - \frac{1}{T}\right)\right] \quad \dots(24)$$

2.3. Model of daily variation of air temperature

The control behavior of photovoltaic converters are ruled by the greatness of the electrical stack, the escalated of sun powered radiation, the encompassing temperature and the wind speed.

For PV found in tall northern scopes, which incorporate most of the domains of Russia, outside meteorological components can have a noteworthy effect on the execution of the SB, and it is

alluring to require them into consideration. The only way is to decide the normal month to month and normal day by day values of the encompassing discuss temperature and wind speed, which are free values from the parameters of the electrical establishment being outlined. The introductory information for their assurance are the factual information of meteorological perceptions, which can be gotten from the chronicles of meteorological locales [14,15].

$$T(t) = \bar{T} + 0,5\Delta T \cos[2\pi(t_{pos} - t_{smax})] \dots(25)$$

Where $\bar{T}$ average daily air temperature, °C; ΔT daily amplitude of air temperature, °C; t_{pos} : local solar time, h; t_{smax} is the local time of the temperature maximum, h; t_s is the period of change in air temperature, h.

2.4.PV module surface temperature model

In expansion to the surrounding discuss, the FM temperature administration is straightforwardly influenced by the radiation vitality coming to the FM surface alongside the sun's beams, and inner warm sources produced due to electrical misfortunes in sun powered cells and contact resistances. The working temperature of the FM amid operation is set as a result of heat trade of its surface with the environment, which features a convective brilliant character. Deciding the current temperature of the FM by hypothetical strategies is essentially outlandish due impact of a wide assortment of components on the method:

warm exchange coefficients and radiation retention coefficients of the module materials, wind quality and course, proportion of the zones of the front and raise surfaces of the module, etc. FM over the surrounding temperature at tall values of sun based insolation can reach 30 °C, so taking under consideration the genuine temperature of the FM beneath working conditions is obligatory. As a result of handling the exploratory information gotten beneath the working conditions of silicon FMs in Siberia, an approximating reliance of the temperature rise of the FM surface on the size of sun based radiation G and the normal wind speed V :

$$\Delta T_{FM} = 0,0283G - 0,0058GV + 0,0005GV^2 \quad 27)$$

Due to the inertia of thermal processes, the PV temperature cannot change instantly, so the model includes an inertial link implemented on the basis of a standard transfer function block.

3. Results and discussions:

In MATLAB, the equations of mathematical model of PV are simulated as shown in Fig.3. It has different blocks connected to each other, representing the equations of the PV model. The schematic diagram has many elements, such as squares, circles, and arrows. Each element represents a part of the PV model, such as a voltage source, a current source, a resistor, or a diode. The arrows show how the electric current flows through the model.

Fig.4 shows the temperature values over time. The graph has four steps, which are numbered as follows: The first step is from 0 to 1 second the temperature is constant at 50 degrees. This means that the solar cell is not exposed to sunlight or heat. The second step is from 1 to 2.5 seconds the temperature increases sharply to 70 degrees. This means that the solar cell is suddenly exposed to a lot of sunlight or heat. The third step is from 2.5 to 3.5 seconds the

temperature decreases back to 50 degrees. This means that the solar cell is no longer exposed to sunlight or heat. The fourth step is from 3.5 to 4 seconds the temperature-0-- increases again to 72 degrees. This means that the solar cell is exposed to sunlight or heat again.

Fig.5 shows the I-V curve, which plots the PV voltage (V) against the PV current (I). Figure 6 shows the PV power curve, which plots the PV voltage (V) against the PV power.

In both graphs, the temperature increases, the current and power of the PV cell decrease. This means that the PV cell is less efficient at higher temperatures.

The reason for this is that the temperature affects the internal resistance of the PV cell. The internal resistance is the opposition to the flow of current within the cell. As the temperature increases, the internal resistance increases, which reduces the current and power output

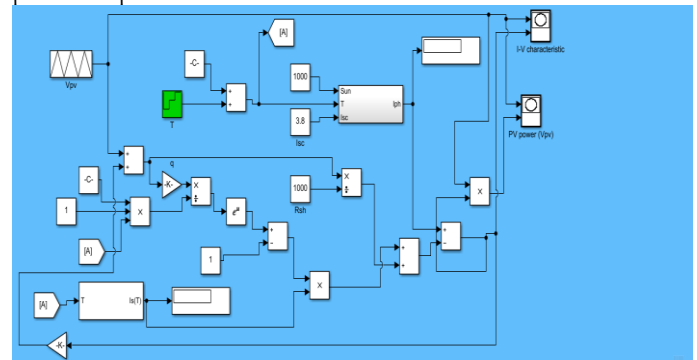


Figure 3. The Matlab simulated PV model.

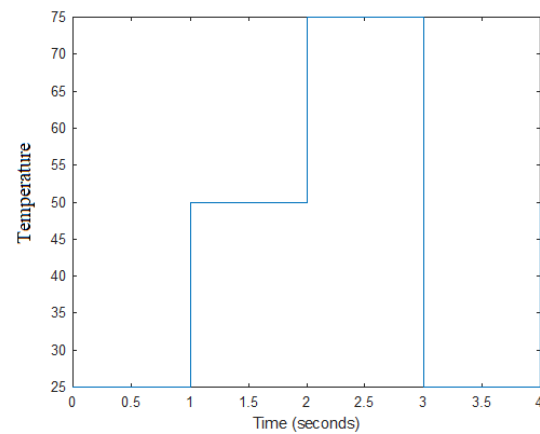


Figure 4. The temperature values of PV.

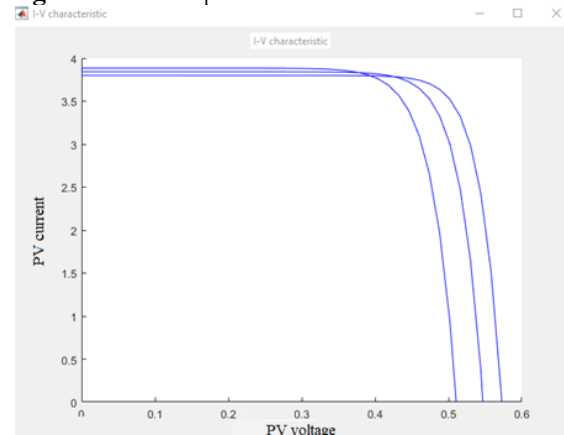


Figure 5. I_V curve.

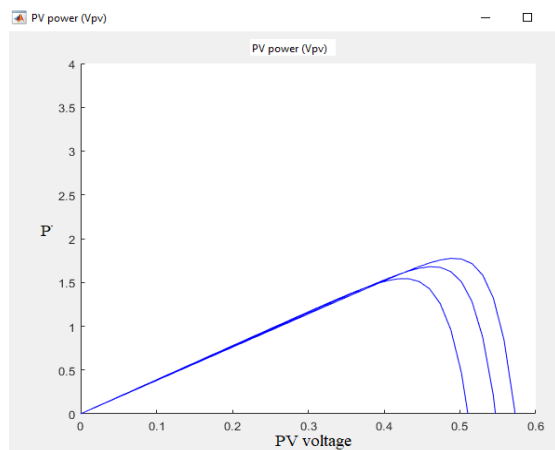


Figure 6. PV power curve.

4. Conclusion

Mathematical models are utilized to calculate solar radiation for PV systems, considering factors like solar radiation incidence and meteorological conditions to determine energy output. Photovoltaic cells convert light into electrical energy through the photovoltaic effect. PV curve construction involves the relationship between output current and voltage, critical for analyzing the performance of solar panels. The PV cell produces less electricity at higher temperatures.

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