Three Dimensional Fuzzy Reliability for System Performance Evaluation

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Abstract

The research proposed a developed methodology for evaluation the system performance in uncertainty associated with traditional modelling methodology is focused on either load L or resistance R variability, but not both. A two-dimensional (2D) fuzzy set (traditional model), represent with the one dimension for universe of discourse (in x-direction) and the second dimension of his membership degree (in y-direction), is not full sufficient to handle both, load and resistance variation of system performance. The theoretical principle basis of this research is based on development of the three dimensional (3D) of fuzzy set that includes system performance variability in load and resistance from two dimensional. The proposed methodology (traditional model) extends the acceptance level of partial performance of system concept to a 3D-dimantion representation. This representation allows to capturing the changing of preferences of decision makers in load and resistance. The major objective of the research is to proposed the original methodology for evaluate system performance and management that is capable of; (a) addressing uncertainty caused by load and resistance variability and ambiguity; (b) integrating objective and subjective evaluation; and (c) assisting system performance management decision making based on a more detailed certainty evaluation of load and resistance variability.

The study proposed two models for fuzzy reliability performance indexes: first traditional model included (I) 2D fuzzy reliability-vulnerability Rv index, (II) 2D fuzzy robustness Ro index; the second developed model (i) 3D fuzzy reliability-vulnerability Rv index, (ii) 3D fuzzy robustness Ro index; and comparing between them. These indexes have the capability of evaluating the operational performance of complex systems. Proposed methodology is illustrated by using the Al-Wathba Water Supply System (WWSS) as a case study**.**

Keywords: Reliability, Fuzzy Set, Fuzzy Reliability, Performance Indexes, Water Supply System.

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النمذجة موثوقية ضبابية ثلاثية الأبعاد لتقييم أداء النظام

كاظم احمد عبد

الخلاصة:

اقترح البحث منهجية مطورة لتقييم أداء النظام في حالة عدم اليقين المرتبطة بمنهجية النمذجة التقليدية التي تركز إما على تباين الحمل L أو المقاومة R ، ولكن ليس كليهما. مجموعة ضبابية ثنائية الأبعاد (نموذج تقليدي) ، تمثل بعدًا واحدًا (في اتجاه x) والبعد الثاني من درجة عضويته (في اتجاه y) ، ليست كاملة بما يكفي للتعامل مع كليهما ، عند تمثيل تباين الحمل والمقاومة لأداء النظام. يستند الأساس النظري لهذا البحث على تطوير لمجموعة ضبابية ثلاثية الأبعاد (D٣) تتضمن تباين أداء النظام في الحمل والمقاومة من بعدين. المنهجية المقترحة (النموذج التقليدي) تمد مستوى قبول الأداء الجزئي لمفهوم النظام إلى تمثيل ثلاثي الأبعاد. يسمح هذا التمثيل بالتقاط تغيير تفضيلات صانعي القرار في الحمل والمقاومة. الهدف الرئيسي من البحث هو اقتراح المنهجية الأصلية لتقييم أداء النظام والإدارة

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القادرة على ذلك ؛ (أ) معالجة عدم اليقين الناجم عن تباين وغموض الحمل والمقاومة ؛ (ب) دمج التقييم الموضوعي والذاتي ؛ و (ج) المساعدة في اتخاذ قرارات إدارة أداء النظام بناءً على تقييم يقين أكثر تفصيلاً لتغير الحمل والمقاومة.

اقترحت الدراسة نموذجين لمؤشرات أداء الموثوقية الغامضة: النموذج التقليدي الأول تضمن (2D fuzzy (I reliability-vulnerability Rv index, 2D fuzzy robustness Ro index(II) . اما النموذج الثاني المطوّر) 3D fuzzy reliability-vulnerability Rv index, (ii) 3D fuzzy robustness Ro (i index ؛ والمقارنة بينها. هذه المؤشرات لديها القدرة على تقييم الأداء التشغيلي للأنظمة المعقدة. يتم توضيح وتطبيق المنهجية المقترحة على مشروع ماء الوثبة في بغداد (WWSS) كدراسة حالة.

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1. Introduction

Since the development of modern technology in the 1960s, the reliability tools are becoming a vital technic in engineering analyses. Therefore, the reliability in probabilistic approaches has extended to achieve the evaluation of systems performance reliability. The probabilistic reliability approach has become very popular tool to address with uncertainty problems.

With the development of technology and science, the researchers gradually appreciate that there is not only random uncertainty, but also a lot of fuzzy uncertainty exists in science and engineering. To solve these problems, the fuzzy theory has been proposed to analyze fuzzy reliability (FR) and becomes a vital tool to treat the systems with fuzzy variables. Based on the fuzzy theory, most of the researches have worked to establish the reliability theory [1].

Today's product and services are facing strong pressure to develop accurate technical tools for evaluation and improving of reliability. Furthermore, there are compressions to improve reliability measure, and total of quality. Therefore, there are necessary matters associated with the problem of reliability evaluation posing a great challenge to the reliability evaluation of systems. The problems are: 1) The probabilistic approach is widely used to handle the problem of reliability of systems. However, the probability usually fails to treat the uncertainty problem that comes with the lack of certainty in, human input, lack of failure data and incomplete record keeping. On the other hand, how to get a probability density function (pdf), especially those that do not have sufficient data failure in their lifetime. 2)Most of the previous studies, especially relating to reliability analysis, depend on the failure data of the system. However, the traditional definition of failure is not adequate, that will present another definition of failure to deal with partial failure. The using of complicated extended algebraic operations in probabilistic approach, there is a real need to develop modeling approaches that can overcome these deficiencies and execution faster than the ones presented in old methods.

1.1. Probabilistic Traditional Reliability Model: In the probability approach, the reliability analysis involves describing load and resistance as related to respective probability distributions [2]. In a result, uncertainty in both, Load L and Resistance R, is introduced using random variables. Thus, the system reliability measured in the terms of probability. The shaded area of the overlap region between the two probability density function of Load $f_L(l)$ and Resistance $f_R(r)$, in Fig.1.The calculation requires the prior knowledge of the pdfs of both, load l and resistance r, and their joint pdfs [3].
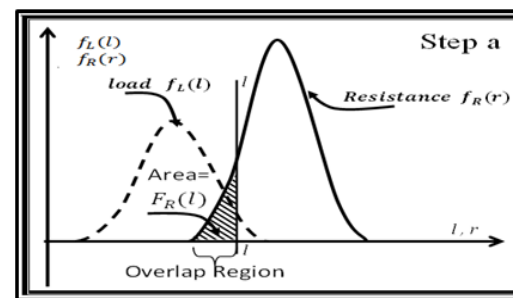

Figure (1): overlap between the pdfs of $f_L(l)$ and $f_R(r)$ [3]

Julwan et al [4]noted that, the major problems in practice, the data is generally insufficient to provide this information. Even if this data is available, the distributions of data are almost estimating by subjectively. The type of distributions of data is always necessary to calculate system reliability.

1.2. FR Modelling Under Fuzzy Assumption: The fuzzy reliability defined by Kapur et al [5] develops the fuzzy binary and fuzzy multi state to fuzzy reliability meaning when the fuzzy sets are substituted by the crisp value sets. Dourado, et al. [6], and Guijie, et al.[7] observed that the fuzzy reliability as defined by Kapure, et al.[5] is more meaningful and realistic.

El-Baroudy, [8], and julwan, et al.[4] proposed fuzzy reliability performance based on the success membership functions for fuzzy reliability modelling by using the fuzzy sets. Moreover, the membership functions of the fuzzy of work events are defined.

Ahmed, [9] addressed fuzzy reliability performance models with the uncertainty in estimating the system reliability. The fuzzy set that the membership functions of the success is defined; it is a meaning way from view point of evaluation the system reliability of performance.

The main objectives of the research are: 1) To develop a model to evaluate the reliability based on performance data, not on failure data as used in the traditional reliability. 2)To Provide the methodology for reliability evaluation of systems while taking into consideration all parameters (Load and Resistance) and does not focus on one only.

This study discovers the usefulness of the fuzzy reliability performance indexes that is suggested by El-Baroudy, [8] and Ahmed,[9] for evaluation of fuzzy reliability performance of system.

2. The Methodology

The methodology Echelons used to develop two models: First Echelon: 2D Fuzzy Reliability Performance Indexes Model in traditional methodology (traditional model). Second Echelon: 3D Fuzzy Reliability Performance Indexes Model in developed methodology (developed model).

2.1. First model: 2D Fuzzy Reliability Performance Indexes

A. **Definition of Fuzzy Failure**: Simonovic,[10] introduced a definition of failure based on the concept of load L and resistance R terms. When the Resistance is less than the Load (R < L) there should be failure. The crisp value identification of failure is not practical and realistic. The partial failure is a very realistic approach. The fuzzy safety margin $\widehat{M}(m)$, represeneted system state, used in measuring system reliability. It is calculated by the fuzzy Resistance $\widehat{R}$ and fuzzy Load $\widehat{L}$ as in Equ.(1). The measure of the system failure or system state, that is:

$$\widehat{M}(m) = \widehat{R} - \widehat{L} \qquad \dots (1)$$

Where: The safety margin $\widehat{M}(m)$ is represented using the system state (SyS).

B. **Acceptable of Performance Level**:

These ambiguous quantities better described by fuzzy theory rather than classical set theory. If the amount of the fuzzy safety margin $\widehat{M}$ is less than m1 then it falls in the complete failure zone and the fuzzy membership function MF is zero in Fig.2 [8]. Similarly, if the value of the safety margin is more than m2 then it will be in the complete safety zone and the value of MF will be one. Any value of the safety margin between m1 and m2 implies that a system is in the zone of fuzzy failure. Where {m1 and m2} $\in \widehat{M}$. [3]
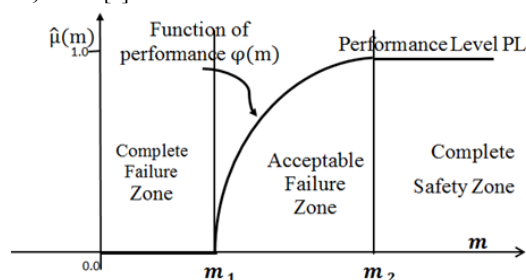

Figure (2): illustration of Fuzzy performance zone [8].

Simonovic [10] defined of the reliability a performance level (PL) measure as represented in Equ.(2):

$$PL = \frac{m_2 * m_1}{m_2 - m_1} \qquad \dots (2)$$

To reflect the subjectively (personally) judgment of the decision maker by different of the performance was assigned by different levels of performance. First level PL1: Rising average of SyS larger than the PL with a High Reliable acceptable performance level; Second level PL2. Average of SyS equal with a Reliable acceptable performance level. Third level PL3: Decreasing boundary of SyS smaller than the PL with an Unreliable acceptable level as shows in Fig.3 [3,12].
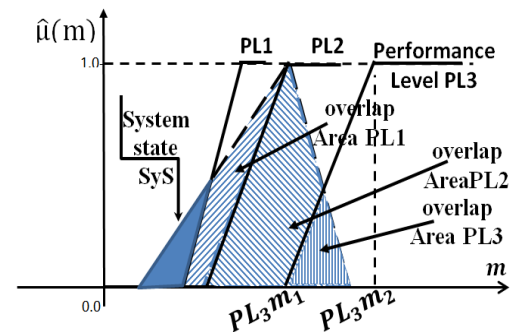

Figure (3): Compatibility system state (SyS) with performance level PL1 MF [3].

C. **Compatibility measure**: purpose of comparing the two fuzzy membership functions is match two fuzzy sets. The defines compatibility as[8,11]:

$$Compatibility \ (Cm) = \frac{Weighting \ overlap \ area}{Weighting \ area \ of \ system \ state}$$
$$= \frac{WOA}{WA} \qquad \dots (3)$$

Where:
Cm : Compatibility between system state (SyS) MF and the performance level PL);
WOA : the weighting zone of overlap between (PL) and SyS MF; and
WA : the weighting zone of the MF of SyS.

D. **Fuzzy Reliability-Vulnerability Indexes**: The index of fuzzy reliability–vulnerability as follows [10,13]:

$$R_v = \frac{\overset{max}{\cdot i \in K} \{Cm_1, Cm_2, \dots . Cm_i\} * PL_{max}}{\overset{max}{\cdot i \in K} \{PL_1, PL_2, \dots . PL_i\}} \ \dots (4)$$

Where:
Rv : the fuzzy reliability-vulnerability index;
PL$_{max}$: PL agreeing to the SyS with max Cm values;
PLi: the i-th PL ;
K: is entire number of the performance level PL.

E. **Fuzzy Robustness Index:** The robustness as a measure of the capability of the system to adjust to an extensive variety of thinkable future load conditions. Redefined this concept in fuzzy environment as the changing in compatibility measure that reflects the changing in future environment conditions. Therefore, the fuzzy robustness R_O index in terms of compatibility measure is[11,14]:

$$R_O = \frac{1}{Cm_1 - Cm_2} \qquad \dots (5)$$

Where: Cm_2 and Cm_1: measures of Cm after and before changing environment in the conditions.

Therefore, the low changing in compatibility measure, leads to the higher value of fuzzy robustness index and the system's ability to adapt to new conditions will also be higher [10].

2.2 The second Echelons:

3D Fuzzy Reliability Performance Indexes Model in developed methodology (developed model) is presented in Fig.4. The 3D approach provides perfect information on variable C and D in both directions (x and y respectively) at each Ci and Dj to improve perfect evaluation of system performance. Due to the significant amount needed for performance data and the length of time for computational modelling of 2D than 3D. The 3D modelling applications become increasingly feasible in system. Because of the rapid development in the use of computers and software in solving calculations to complex problems. The developed model is generated from the traditional model to evaluate the performance of the system in three dimensions by proposing two fuzzy reliability performance indexes: (I)3D Reliability-Vulnerability index and (II)3D Robustness index.

A. Identify model variables and prepare data: Capacity variable C_i in x axis and Demand variable D_j in y axis and the third dimension is degree of membership function $\hat{\mu}\left((C_i, D_j)\right)$ as in Fig.4. For membership degree $\hat{\mu}_{(ci)}$ for Ci variations. The dimension Performance Levels (PL) is variable, selected subjectively based on average system state.
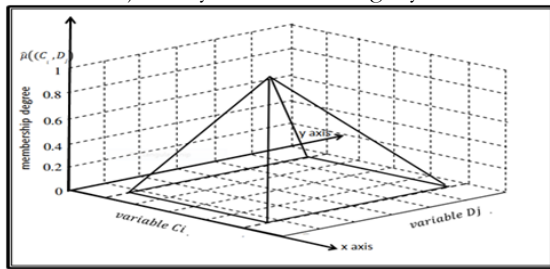

Figure (4): 3D of modelling C_i and D_j

C. Fuzzy Set for joint the model of component performance: The 2D fuzzy sets developed for; i) $\underline{C}$ variability of performance, and ii) $\underline{D}$ variability of performance are used for capturing various sources of Ci and Dj uncertainty, respectively. The developed process based on Fig.5 the 3D joint MF of the system performance provides values of the Ci and Dj variability in system performance.
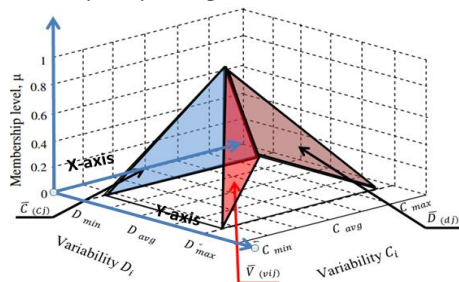

Figure (5): 3D joint fuzzy set of Ci and Dj

The 3D joint MF of values of the Ci and Dj values used to computation of Ci and Dj variability profiles in three dimensions and permits the accurate

computation of the system performance and offers a best prediction of the outcomes of Ci and Dj variability processes. The definition of the 3D joint fuzzy set is given as follows:

$$\underline{V} = \left\{ (c_i, d_j), \mu_{\underline{V}}(c_i, d_j) \mid \forall c_i \in C, d_j \in D \right\}$$

$$0 \leq \mu_{\underline{V}}(c_i, d_j) \leq 1 \qquad \dots (6)$$

Where:

$\underline{V}$: denotes 3D fuzzy set for C_i and D_j variability;

$\underline{C} = C_i$: the capacity performance;

$\underline{D} = D_j$: the demand performance ; and

$\mu_{\underline{V}}$: degree membership of the 3D fuzzy set $\underline{V}$.

3.2.3.Definition of 3D Performance Level: The extend 2D model in Fig.1 and Equ.(2) to 3D model represented in Fig.6 and Equ.(7). The 3D performance level (PL) is represented as 3D fuzzy MF $\widehat{PL}(v)$, based on the system state value v for Ci and Dj in Fig.6. The $\widehat{PL}(v)$ is defined as follows:

$$\widehat{PL}(v_{ij}) = \left\{ 1 \; \varphi_k(v) 0 \; if \; v \leq v_1 \; if \; v \in [v_1, \; v_2] \; if \; v \geq v_{ij\,2} \right\} \dots (7)$$

Where:

$\widehat{PL}\left(v_{ij}\right)$: The 3D Fuzzy MF of performance levels PL ; v_1 and v_2: lower and upper bounds of the PL; $\varphi_k(v)$: Functional of the levels of PL: K;(1, 2, 3): the subjective levels PL MF.
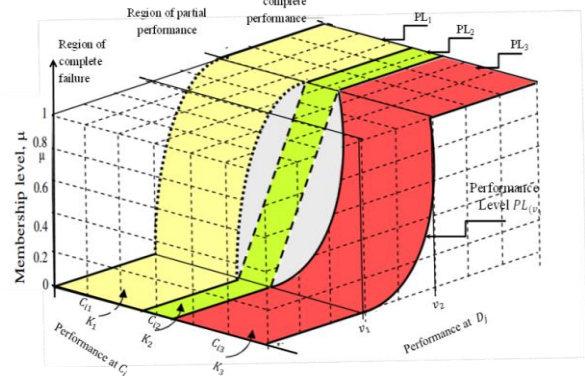

Figure (6): 3D Fuzzy representation of Ci and Dj variability and PL.

Subjectivity of the $\widehat{PL}$ (in Equ.(2)) with different levels of the system performance are assessed by subjected the values of (v1 and v2) by personal interviews with experts, selected subjectively based on average system state ($\underline{V}_{avg}$) MF value. As shown in Fig.6 the process selected is:

1-$\widehat{PL}1$ at Reliable level K1: The MF of PL value selected subjectively where the boundary of the average system state ($\underline{V}_{avg}$) larger than PL MF value. That means system performance is in Reliable level.

2-$\widehat{PL}2$ at Neutral level K2: The MF of PL values selected subjectively where the boundary of the average system state($\underline{V}_{avg}$) MF is equal to PL

value. That mean the system performance is in Neutral level.

3-$\widehat{PL}3$ at Unreliable level K3: The MF of PL values selected subjectively where the bounds of the average of system state (V_{avg}) smaller than PL MF value. That means the system performance is in Unreliable level.

D. Dimensionality Reduction Operation: Dimensionality Reduction (DR) Operation is an important task in 3D fuzzy, it facilitates classification, compression, and visualization of high-dimensional of data by justifying the curse of the dimensionality and extra undesired properties of high-dimensional (3D) spaces. It is the conversion of high-dimensional (3D) data into a meaningful formation of compact dimensionality (2D). Theoretically, the compact formation has a dimensionality that relates to the real dimensionality of the data. This process needed the minimum number of parameters to account for the observed properties of the data[9,11]. The 3D Fuzzy input is transformed by DR into a traditional fuzzy output as shown in Fig.7.
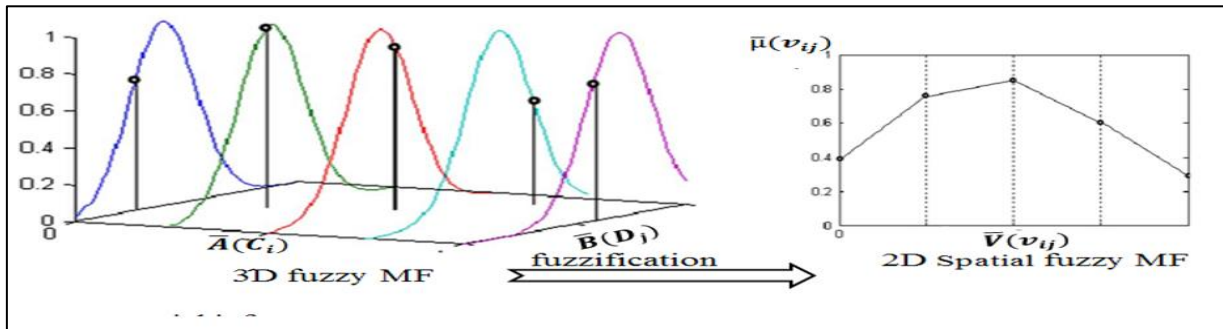

Figure (7): 2D Spatial output information translate from at each crisp input.[9]

The weighted and centroid methods are most frequently used in fuzzy applications since they are the most computationally efficient methods. As shown in Fig.8, the 3D fuzzy set $\underline{V}$ is regarded as a 2D MF on the plane ($\underline{C}$ and $\underline{D}$).
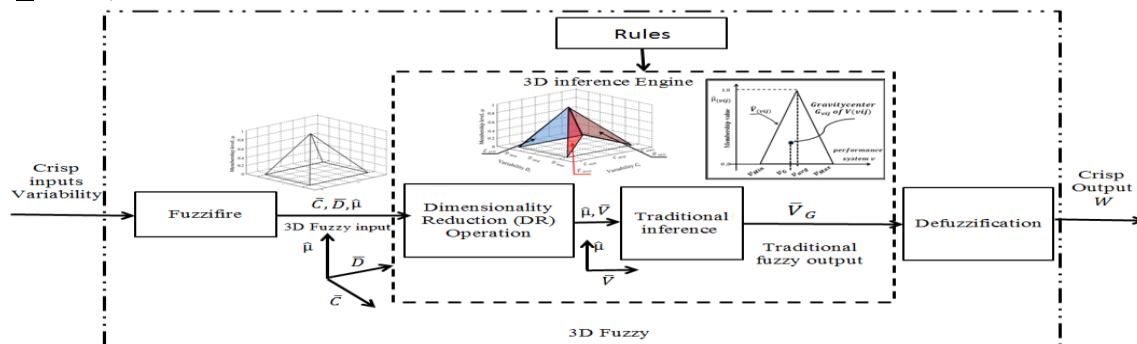
Figure (8): Dimensionality Reduction Operation of 3D fuzzy.

E. **The 3D Fuzzy Reliability Performance Compatibility**: The basis for reliability evaluation is comparative analysis of two membership functions as shown in Fig.9: (a) system performance MF of $\underline{V}(v)$ in Equ.(6) and (b) the predefined of partial performance level MF in Equ.(7).

Fuzzy Compatibility measure Cm is based on that an overlap in the zone (area) of high is preferable to an overlap in low zone membership values. Therefore, the fuzzy compliance proceeds into account the Weighting area approach. The 3D fuzzy compatibility measure in Equ.(8) is:

$$Cm_f = \frac{WOA_{ijf}}{WA_{ij}} \quad \dots (8)$$

Where:

Cm_f: the compatibility of f-th performance levels $\widehat{PL}$;

WOA_{ij}: the weighted overlap area between MF $\underline{V}$ and f-th of MF $\widehat{PL}$

WAij : the weighted of MF $\underline{V}$ for Ci and Dj; and

K= f =3: the number of performance levels.

Figure (9) : Overlap area between performance MF $\hat{V}(v)$ and of performance *level PL* MF.

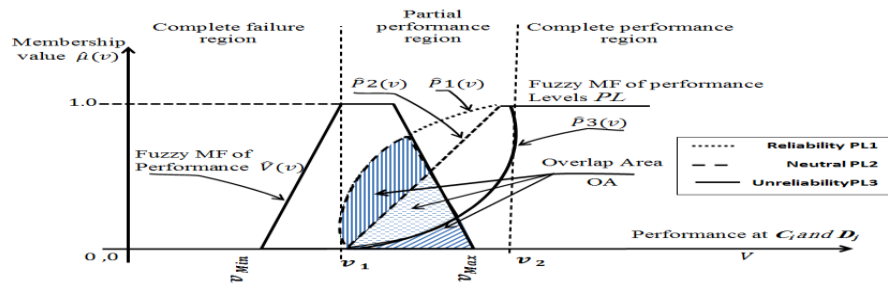**F. The 3D Fuzzy Reliability Performance Indexes:**

- The 3D Fuzzy Reliability-Vulnerability Index: Fuzzy Reliability-Vulnerability Rv_{ij} index for system performance reliability evaluation is calculated using:

$$Rv_{ij} = \frac{\{Cm_1, Cm_2, \dots. Cm_{ij\,f}\} * \widehat{PL}_{max}}{\{\widehat{PL}_1, \widehat{PL}_2, \dots. \widehat{PL}_f\}} \quad \dots (9)$$

Where:

Rv_{ij} : 3D fuzzy Reliability-Vulnerability index for Ci and Dj;

$\widehat{PL}_{max}$: performance level agreeing to the max compatibility value for Ci and Dj;

PL_f : performance level of the f-th level;

$Cm_{ij\,f}$: the fuzzy compatibility measure between f-th performance levels and system performance $\underline{V}$.

- The 3D Fuzzy Robustness Index model

The adaptableness of the system performance to the changing in performance levels is in both Ci and Dj variability. Two compatibility measure values are used as inputs in the following Equation:

$$Ro_{ij} = \frac{1}{Cm_{ij\,1} - Cm_{ij\,2}} \quad \dots (10)$$

Where:

Ro_{ij}: 3D fuzzy Robustness index for Ci and Dj ;

Cm_{ij1}: the compatibility measure before the change in the performance levels; and

Cm_{ij2}: the compatibility measure after the change in the performance levels.

G. Input data of Capacity and Demand of Components

Table (1): summarized input data for ThWSS

System	Flow capacity			Demand								
	Average daily	Planned capacity	Designed capacity	Flow			Chlorine			Alum		
				Average	Max	Min	Average	Max	Min	Average	Max	Min
ThWSS	28000	48000	76800	28000	35000	13000	478	700	256	750	1300	375

Tables (1): are prepared and treated to be used as the input data of Capacity(C) and Demand (D) data for flow of raw water, alum and chlorine for case study.
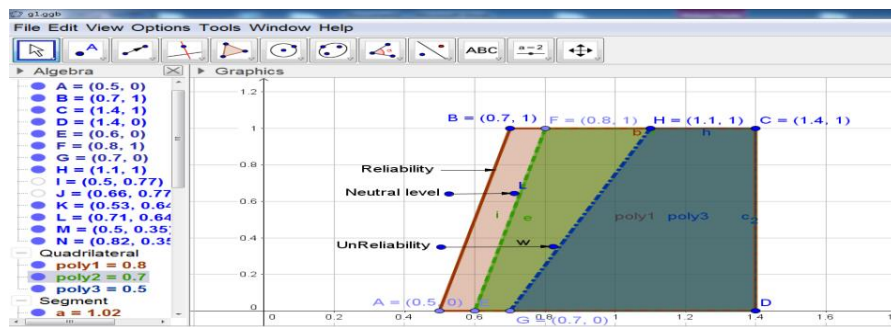
Figure (10): 2D represent the Acceptable performance levels

3. Application of fuzzy reliability
3.1 Subjectivity of performance level PL MF values:

The selected process is based on average system state of entire system $(SyS\,m_{avg})$ MF value. From Table (1) the $(SyS\,m_{avg})$is (0.8). Therefore, The MF of performance levels (PL) defined by using four points of TpMF PL1=(0.5,0.7,1.4,1.4), pL2=(0.6,0.81.4,1.4), and PL3=(0.7,1.11.4,1.4) as

shown in Fig.10. The three selected values are selected to reflect three different views of decision makers as defined by the reliability performance measure in Equ.(2). The reliability of (PL) is (1.75, 2.4 and 1.92). The results of reliability are different with high variants. Those mean the small change in uncertainty caused a large influence on the reliability system performance.

3.2. Determine the area and weighted:

The calculation of Cm is dependent on the calculation of area and its weight by using Table (1) Equs.(8),(9) for Tg and Tp MFs, respectively. By using GeoGebra program to draw and determine the area and overlap area. For example: For Tg, from Table(1) the three values of TgSyS (m_{min} 0.0, m_{avg} 0.8, m_{max} 1.66) and of TpSyS (0.0, 0.55,1.11,1.66) of system(in Table (1) upper row left side) and the

values of PL, using GeoGebra program to draw and determine the area and overlap between system(SyS) and PLs shown in Fig.11.

Fig.11 shows the overlap areas of the SyS MF with different PL for Tg and Tp MF for only ThWSS. The overlap areas and weights of the SyS MF with different PL of all component performance for Tg and Tp MF are determined and listed in Tables (2) for each components system of both ThWSS .
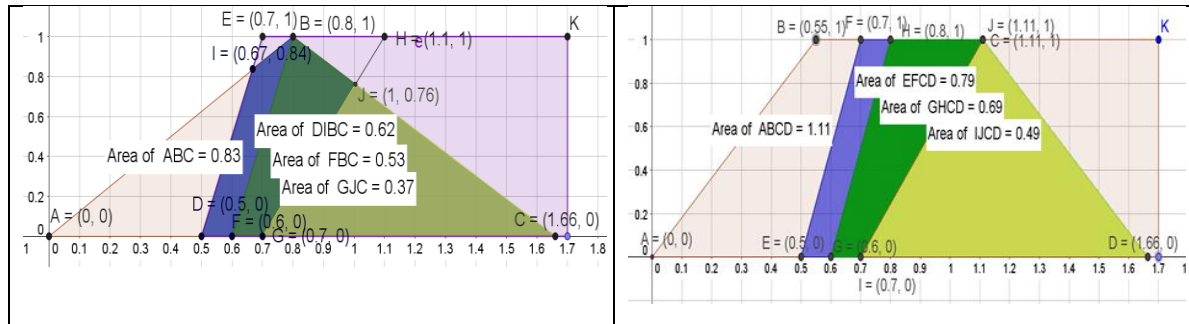

Figure. (11): The draw and determine area and overlap area by GeoGebra program

Table (2): Example of results of area and overlap area for Tg&Tp MF

ThWSS	2D Tp Tringle													
	System state						Overlape					indix		
	a	b	c	Area	Gc	Weight	d	F	Area	Gc	Weight	Cm	Rv	Ro
System	0.00	0.800	1.66	0.830	0.82	o.681								
PL1							o.40	0.67	0.62	0.91	0.56	0.827	0.603	30.866
PL2							0.50	0.50	0.53	1.02	0.54	0.794		
PL3							0.80	0.01	0.37	1.12	0.42	0.611		

ThWSS	2D Tp Trapezoidal															
	System state							Overlape						indix		
	a	b	c	d	Area	Gc	Weight	E	K	F	Area	Gc	Weight	Cm	Rv	Ro
System	0.00	0.550	1.11	1.66	1.11	0.83	o.921									
PL1								o.40	0.70		0.79	0.97	0.76	0.830	0.605	14.80
PL2								0.50	0.80		0.69	1.02	0.70	0.762		
PL3								0.80	1.10		0.49	1.17	0.57	0.621		

Compatibility Measure Cm: to determine Cm measure using Table (2) and Equ.(8).

$$CM_{Sys} = \frac{WOA}{WA}, CM_1 = \frac{0.56}{0.681} = 0.827,$$

$$CM_2 = \frac{0.54}{0.681} = 0.794,$$

$$CM_3 = \frac{0.42}{0.681} = 0.611.$$

To calculated the 2D Fuzzy Reliability Performance Indexes Rv and Ro of Tg MF.

$$Rv_{systm} = \frac{\overset{max}{\cdot i \in K} \{CM_1, CM_2, \dots CM_i\} * LP_{max}}{\overset{max}{\cdot i \in K} \{LP_1, LP_2, \dots LP_i\}},$$

$$Rv_{systm} = \frac{0.82 * 1.75}{2.4} = 0.60$$

$$Ro_{system} = \frac{1}{CM_1 - CM_2} = \frac{1}{0.827 - 0.794}$$
$$= 30.87 \ . \ .$$

The overall components system for Rv and Ro indexes are calculated and the result listed in Table (3) for Tg and Tp MF.

Table (3): results of overall for Rv and Ro indexes

ThWSS	2D Triangle		2D Trapezoidal	
	Rv	Ro	Rv	Ro
System	0.60	30.9	0.600	14.8
Sub System				
Intak	0.93	71.8	0.74	48.3
Low lift pumping	0.72	10.9	0.88	16.1
Rapid mix	0.58	10.6	0.56	18.5
Flocculation	0.70	82.3	0.88	20.1
Sedimentation	0.54	14.1	0.54	11.4
High lift pumping	0.93	71.8	0.74	48.3
Pressurized filter	0.67	57.2	0.69	14.3
Chlorine	0.86	13.3	0.74	44.2

3.3. Analysis of 2D Fuzzy Reliability Indexes

Through results of 2D Fuzzy Reliability Indexes it is showing 2D Fuzzy Reliability used in the evaluated systems performance. Table (3) for Rv , Ro show that:

1. From Table (2) the selected process of (m_1,m_2) for different performance levels represented different levels of uncertainty. The results (1.75, 2.4 and 1.92) are different with high variants. Those mean the small change in uncertainty caused a large influence on reliability system performance.

2. Weakness component is rapid mixing ($TgRv$ 0.58, $TpRv$ 0.56) and sedimentation ($TgRv$ 0.54, $TpRv$ 0.54). This will be an incentive to develop a methodology to assist the decision maker to aid the decision process using developed models.

3.4. The 3D Fuzzy Reliability Performance Indexes Model (developed methodology)

The partitioning fuzzy sets of Tg and Tp MFs are calculated by using Equs.(8) and (9), respectively. From the last right column in Table (3), The results of partitioning process of Tg and Tp MFs are listed in Table (4) These results are used to determine WA and WOA .

Table (4): The result of partitioning process of Tg and Tp

ThWSS	Tp Cap			Tg Cap				Tp Reg			Tp Reg				Tp sys			Tg sys			
	S	M	G	S	M1	M2	G	L	N	H	L	N1	N2	H	V	W	Z	V	W1	W2	Z
System	0.34	0.92	2.00	0.34	0.89	1.45	2.00	0.30	0.69	1.19	0.30	0.60	0.89	1.19	0.00	0.80	1.66	0.00	0.55	1.11	1.66
Sub System																					
Intak	1.48	2.52	4.00	1.48	2.32	3.16	4.00	0.68	1.48	1.80	0.68	1.05	1.43	1.80	0.00	1.04	3.32	0.00	1.11	2.21	3.32
Low lift pumping	1.58	2.68	4.00	1.58	2.39	3.19	4.00	0.82	1.51	1.97	0.82	1.20	1.59	1.97	0.00	1.17	3.18	0.00	1.06	2.12	3.18
Rapid mix	0.34	0.92	2.00	0.34	0.89	1.45	2.00	0.30	0.69	1.19	0.30	0.60	0.89	1.19	0.00	0.23	1.70	0.00	0.57	1.13	1.70
Flocculation	3.33	4.17	5.00	3.33	3.89	4.44	5.00	1.53	3.33	4.00	1.53	2.35	3.18	4.00	0.00	0.84	3.47	0.00	1.16	2.31	3.47
Sedimentation	1.33	1.67	2.00	1.33	1.55	1.78	2.00	0.60	1.33	1.63	0.60	0.94	1.29	1.63	0.00	0.34	1.40	0.00	0.47	0.93	1.40
High lift pumping	1.48	2.52	4.00	1.48	2.32	3.16	4.00	0.68	1.48	1.80	0.68	1.05	1.43	1.80	0.00	1.04	3.32	0.00	1.11	2.21	3.32
Pressurized filter	3.70	6.30	10.0	3.70	5.80	7.90	10.0	1.69	3.13	4.56	1.69	2.65	3.60	4.56	0.00	3.17	8.30	0.00	2.77	5.53	8.30
Chlorine pumps	1.46	2.50	4.00	1.46	2.31	3.15	4.00	0.67	1.25	1.82	0.67	1.05	1.44	1.80	0.00	1.25	3.30	0.00	1.10	2.20	3.30

A. **Definition of 3D Performance Level**:

The fuzzy sets for performance system level Tp MFs is calculated by Equ.(2). The selected process is based on average system state of entire system (V_{avg})MF value from Table (4) the (V_{avg} 0.8). Therefore, subjected the values of (v_1 and v_2) is defined; as PL$_1$=(0.4,0.7,1.4,1.4), k$_2$=(0.6,0.8,1.4,1.4), and k$_3$=(0.7,1.1,1.4,1.4).

Table (5) the example results of weighting area WA_{ij} and weighting overlap area WOA_{ij} of system for both shapes (triangle and trapezoidal) MF for each subsystems of ThWSS.

The C_m Measure is calculated by using Equ.(8). To calculate the two 3D Fuzzy Reliability Performance Indexes Rv and Ro of triangle MF by using Equ. (9 and 10) :

$$CM_{System} = \frac{WOA_{ij}}{WA_{ij}}$$

$$CM_1 = \frac{0.322}{0.716} = 0.45$$

$$CM_2 = \frac{0.372}{0.772} = 0.482$$

$$CM_3 = \frac{0.522}{0.957} = 0.545 \ .$$

$$Rv_{systm} = \frac{[CM1..CMn] * Lp_{ijmax}}{[Lp1..Lpn]}$$
$$= \frac{0.545 * 1.925}{2.4} = 0.438$$

$$Ro_{system} = \frac{1}{CM_1 - CM_2} = \frac{1}{0.45 - 0.482}$$
$$= -31.11$$

Table (5): summarized the result of (WOA_{ij} and WA_{ij}) and (Rv and Ro) for Tg and Tp

ThWSS	System state	3D Tp Triangle				System state	3D Tp Trapezoidal			
		Overlape		indix			Overlape		indix	
System	Weight	Weight	Cm	Rv	Ro	Weight	Weight	Cm	Rv	Ro
PL1	0.716	0.322	0.450		-31.11	0.551	0.322	0.584	0.426	72.26
PL2	0.772	0.372	0.482		-15.73	0.652	0.372	0.571		46.18
PL3	0.957	0.522	0.545	0.438	-10.45	0.951	0.522	0.549		28.17

Table (6): summarized the result of Rv and Ro of each subsystem of ThWSS system.

ThWSS	3D Triangle		3D Trapezoidal	
	Rv	Ro	Rv	Ro
System	0.438	31.11	0.426	72.26
Sub System				
Intak	0.447	44.49	0.442	68.44
Low lift pumping	0.526	27.28	0.568	7.48
Rapid mix	0.438	31.11	0.426	72.26
Flocculation	0.440	31.11	0.428	79.18
Sedimentation	0.446	46.18	0.540	39.56
High lift pumping	0.447	36.21	0.443	67.16
Pressurized filter	0.448	44.49	0.444	43.74
Chlorine	0.448	44.26	0.442	64.84

B. **Analysis of 3D Fuzzy Reliability performance Indexes:** Through results of 3D Fuzzy Reliability Indexes it is showing that 3D Fuzzy Reliability used in the evaluated of systems performance. Table (6) for Rv , Ro shows that**:**

1. For each subsystem for 3D Tg and TP shapes, the values of 3DRv index are more similar, (i.e. Tg Rv 0.48, 0.52, 0.44, 0.44, 0.44, 0.44,0.45, 0.45) That mean the 3D Fuzzy Reliability Indexes is better than 2D Fuzzy Reliability Indexes for using to evaluate system performance because the results are more realistic than 2D. Also these results indicate that using Tp MF give better results than Tg MF shape. This supported the results in 2D Fuzzy Reliability Performance Indexes too.

2. The weakness component is the subsystem rapid mixing (Tg Rv 0.438 and Tp Rv 0.426). The Cm_{max} of the subsystem rapid mixing overlaps with the reliable level of performance (PL$_1$). This indicates that the designed capacity was at the level of performance reliability, but its performance is not good. This will be an incentive to develop a methodology to assist the decision maker to aid the decision process using developed models, also to help decision maker to select a better performance level.

C. **Significance of system Components**

For the Tg and Tp MF shape, shown in Table (6), the subsystem rapid mixing is the weakest part in system. The SyS MF changes significantly with addition of one component from the Alum mixer of rapid mixing subsystem. Enhancement of Alum mixing elements will lead to the enhancement of entire system performance.

Table(7) summaries results of the fuzzy performance index for both cases before and after changing the Alum mixer component MF value. The fuzzy reliability-vulnerability R_v index increased from 0.42 to 0.60, which means an increase of 43% . On the other hand, the fuzzy robustness R_o index increased from 71 to 75.5 indicating an improvement in system robustness.

Table (7): Fuzzy Reliability Performance Indexes change due to the improvement of Rapid mixing subsystem.

Indexes	Before change	After change
Rv	0.42	0.60
Ro	71.0	75.5

Change in designed capacity of the critical component and consequently its MF results in the appearance of new critical components that control entire system performance.

4.Conclusions:

The study proposed two models for fuzzy reliability performance indexes: (i) 2D and 3D fuzzy combined reliability-vulnerability Rv indexes, (ii) 2D and 3D fuzzy robustness Ro indexes; and comparing between 2D and 3D. These indexes have the capability of evaluating the reliability performance of complex systems. The proposed indexes provide a modeling uncertainty in reliability problems. The methodology analysis based on performance data (i.e. design Capacity and average Demand) without using failure data (as in traditional reliability measure). The 2D and 3D models suggested in this study demonstrate performance consistent with expectations. Where the 3D of (TpRv 0.44) is more realistic than the 2D of (TpRv 0.60), because of the data performance shows the system has not greater efficiency in system performance. The 3D model (i.e. $TgRv$ 0.44, $TpRv$ 0.44) is not significantly sensitive for shape of MF, which it turn reduces the effect of subjectivity in the decision making process. The selected subjectively three performance levels will assist the decision maker to select better performance level.

5. References

[1]. S. A Mutlag., Dawood H. K. "A Maintenance Management -based Decision Support System for Multicriteria using AHP Methodology for Power Plants" 82(5) (2020) PP.10146-10153
[2]. T. Hashimoto,"Reliability, Resiliency, and Vulnerability Criteria For Water Resource System Performance Evaluation". Cornell University, Ithaca, New York, 5 (1986) PP.10-25.
[3]. S. A Mutlag. and A. H. Kassam. "Fuzzy Reliability-Vulnerability for Evaluation of Water Supply System Performance" Anbar Journal Of Engineering Science, Vol 8, 2019 PP.72 – 82,
[4]. H.P.Julwan, L. Jie, Z.Guangquan, P. Witold,"A Fuzzy Reliability Assessment of Basic Events of Fault Trees Through Qualitative Data Processing", Elsevier B.V.243 (2014) PP.50-69.
[5]. C.K.Kapur,L.Zhaojun,"Models and Measures for Fuzzy Reliability and Relationship to Multi-State", Reliability International Journal Performability Engineering ,7 (2011) PP.241 – 250.
[6]. A. D. Dourado., Lobato, Cavalini F. S., A. A., & V. Steffen, Fuzzy reliability-based optimization for engineering system design. International Journal of Fuzzy Systems, 21(5), (2019) PP. 1418-1429.
[7]. L. Guijie, L. Zhenzhou, X. Jia, "A Fuzzy Reliability Approach for Structures Based on the Probability Perspective". Elsevier, Structural Safety, 54 (2015) 10-18.
[8]. I. El-Baroudy, & S. Simonovic,"Fuzzy Criteria for the Evaluation of Water Resource Systems Performance". Jornal of Water Resources

Research. American Geophysical Union AGU JOURNAL, 40.10(2006),PP.10-20.

[9]. S. Sh. Ahmad,"Spatial and Temporal Risk Assessment for Water Resources Decision Making" Ph.D. Dissertation, Department of Civil and Environmental Engineering , University of Western Ontario London, Ontario, Canada, 2010, pp. 64-75.

[10]. S. Simonovic, "Probabilistic Assessment of Power System". Univercity Manchester Electrical Energy, Manchester, U.K. Proceedings of the IEEE,. 88(2009) PP.140-162.

[11]. H. Kaplan, & M. Can, "Fuzzy Reliability Theory in the Decision-Making Process". In Advancements in Fuzzy Reliability Theory ,2021. (pp.76-89).

[12]. K.Sruthi, & M.Kumar, (, March). Fuzzy reliability estimation of a repairable system based on data with uncertainty. In AIP Conference Proceedings (Vol. 2336, No. 1, 2021 p. 040011). AIP Publishing LLC.

[13]. P.Kumar, & S.B.Singh, "Fuzzy reliability of systems using different types of level (λ, 1) interval-valued fuzzy numbers". Reliability Engineering Theory and Applications, CRC Press Taylor & Francis Group, (2018). PP.197-213.

[14]. S. A Mutlag."Decision Making in Materials Selection: an Integrated Approach with AHP." Anbar Journal for Engineering Sciences 7.(4), 2019,PP.399-407.