



Effect of Crack Length on Stresses in a Plate with a Hole

Thaker Saleh Dawood¹, Younis Khalid Khdir^{2*}

Authors affiliations:

1) Dept. of Basic Sciences,
College of Agricultural
Engineering Sciences,
University of Duhok, Kurdistan
Region-Iraq.
thaker.saleh@uod.ac

2*) Dept. of Mechanical and
Energy, Erbil Technical
Engineering College, Erbil
Polytechnical University,
Kurdistan Region-Iraq.
younis.khdir@epu.edu.iq

Paper History:

Received: 7th Oct. 2021

Revised: 30th Nov. 2021

Accepted: 30th Jan. 2022

Abstract

The field of mechanics concerned with studying the propagation of cracks in materials is Fracture Mechanics. Technology systems are meant to withstand the loads to which they are likely to be exposed when in use. Material imperfections arising at the time of production or use of the material are, however, unavoidable and must therefore be taken into account. A stress intensity factor is a fracture parameter that defines the part failure. This paper study's the effect of cracks on the stresses of rectangular plates having a hole in the centre. The plate was subjected to tensile pressure at the top side while maintaining the bottom side fixed. The plate had four cracks distributed around the central hole at 45° on each side. The effect of the length of the cracks on the resulted stresses and strains was investigated. Also, the effect of the position of the crack on the resulted stresses and strains was studied. Finite element models for the different plate cases were built using ANSYS software. The results showed that increasing the crack length resulted in to increase in the stresses and strains. The dimensions of the plate width, height, and thickness were 150 mm, 300 mm, and 1 mm respectively, and the crack position was investigated for different crack lengths (5, 10, 15, 20, 25 mm) however the results were not steady as it looks that the crack lengths have changed the stress distribution over the plate.

Keywords: Stress Intensity Factor, Crack Length, Plate, Hole, Finite Element Method.

تأثير طول الشق على الاجهادات في لوح فيه ثقب

ذاكر صالح داود، يونس خالد خدر

الخلاصة:

في هذا البحث تمت دراسة انتشار التشققات في المعادن والتي تتعرض الى اجهادات اثناء الاستخدام حيث لا يمكن تجنب العيوب المادية التي تظهر في وقت انتاجها او استخدامها وبالتالي يجب الحذر بنظر الاعتبار عامل شدة الاجهاد (SIF) والذي يعتبر العامل الرئيسي الذي يحدد الفشل. تتناول هذه الدراسة تأثير التشققات على الخواص الميكانيكية للألواح المستطيلة التي تحتوي على ثقب في المنتصف. حيث تتعرض اللوحة لضغط الشد في الجانب العلوي مع الحفاظ على الجانب السفلي ثابتاً. تحتوي اللوحة على أربعة شقوق موزعة حول الفتحة المركزية عند 45 درجة في كل جانب. تمت دراسة تأثير طول الشقوق موضعها على الاجهادات والانتعالات الناتجة. تم بناء نماذج العناصر المحدودة للألواح المختلفة باستخدام برنامج ANSYS. وأظهرت النتائج أن زيادة طول الشق أدى الى زيادة الاجهادات. تم فحص موضع الشق لطول مختلفة (5، 10، 15، 20، 25 مم) ولكن النتائج لم تكن ثابتة حيث تبين أن أطوال الشقوق قد غيرت توزيع الاجهادات على الصفيحة.

1. Introduction

Residual stresses interfere with stresses generated through applying the load in a complex manner in an engineering part comprising a crack. Usually, to predict the probability of fracture, computational techniques are needed, to produce results that are unique to the range of conditions under study. The analysis under discussion here can be considered as an effort to

explain the overall behaviour of the crack for a specific distribution of the residual tension, loading, and geometry to provide insight into the effects of more complicated analyses.

The application of a plate with a hole has many applications in the industry therefore many studies investigated the plate with a hole. Kumar et., al. (2021) Tensile characteristics of a hybrid kevlar-glass



reinforced epoxy composite with many holes were assessed., They analyzed the experimental findings to determine the best location for the holes. The researchers also want to see if there is a link between the pitch, diameter, and size of the smaller and bigger holes in the specimen. [1]. Mekalke and Kavade (2012) examined the mesh quality of the resulted stresses in their work on a plate with a hole undergoing uniform tensile force [2].

The thickness of the plate affects the mechanical characteristics. Gujar and Ladhane (2015) showed that the deflection decreases with increasing the plate thickness in circular plates. The application of uniform load types can increase the agreement of the results when comparing theoretical and FE simulation records [3]. The effect of plate thickness and plate diameter of the hole on the stresses and deformations were studied by Dhanjal and Arora (2015) ANSYS FE package was used to simulate the model cases. Deformation decreases with increasing thickness. The thickness to diameter ratio has no effect on the occurrence of the maximum stresses at the edge of the hole [4].

Crack propagation has been studied due to its effect on the plate's health. A crack propagation investigation in a plate with a hole was performed by Chorfi and Necib (2016) assuming the hole as a crack initiation case [5]. FE models were used to simulate the study. Dongdong Jiang (2020) investigated the two-dimensional issue of a circular void in infinite thermoelectric materials with two unequal cracks while exposed to a uniform electric current and thermal flux [6].

Bowie (1964) Showed that the Mapping function can be utilized to explore the effect of crack length and its relation to the width and height in rectangular tensile sheets [7]. Extending from Bowie's work, Kutter (1970) classified the effect of the crack length on the critical pressure [8]. He concluded that the crack length has no effect on the critical pressure for crack lengths longer than twice the hole radius. While the effect of the crack length of the critical pressure increases for crack lengths less than twice the hole radius. P. WeiBgraeber et al (2016) studied a closed type empirical finite fracture mechanics technique is used to investigate crack initiation at elliptical holes in plates under uniaxial stress [9]. Stress intensity factors (SIFs) from the literature are addressed and linked to comprehensive numerical data to allow for a thorough analysis of crack initiation. An enhanced stress intensity factor (SIFs) approach is suggested based on physical reasoning that demonstrates excellent alignment with numerical findings for a broad variety of ellipse stress concentration factors and cracks lengths. An optimal Finite Fracture Mechanics solution is achieved using the exact solution of the stress field in the notched plate and the suggested stress intensity factor. Wei Yi et., al (2020) studied the several crack-hole problems under both remote and surface pressures, a modern integral equation approach focused on elementary solutions and they proposed superposition theory [10]. Many crack and hole cases are solved to correlate the proposed method's measurement findings of interacting stress

strength factors (SIFs) in comparison to those produced using the Green's function technique, the dislocation density method, the finite element method, and the digital photo-elasticity method., as well as to examine the influence of loading conditions and crack-hole geometric parameters. The findings of the research indicate that when $x \geq y$, there is a neutral inclination angle θ of the crack (with x-axis) that prevents the crack-hole interaction. Tao Chen et., al (2020) Experiment and the finite element method (FEM) were used to evaluate the fracture propagation and fatigue life of CFRP repaired steel plates with multi-holes [11]. The results of the tests indicated that the existence of multi-holes reduced the steel plate's fatigue life, but that after being repaired with CFRP, the fatigue life was substantially improved. The number and placement of multi-holes had a substantial impact on fracture propagation trajectory and fatigue life, with the number and location of multi-holes being the most influential factors. The adjustment of the stress intensity parameters was mostly responsible for the influence of multi-holes on crack development. (SIFs), according to more finite element research. Residual stress can affect the magnitude of the applied stresses in plates undergoing cracks. Wu et., al (2019) studied the elastic and elastic-plastic behaviour of plates having cracks in the center and exposed to uniaxial loads [12]. Stress intensity factor and different lengths were used to examine the elastic behaviour while the strip yield model is utilized to study the elastic-plastic behaviour. The stresses were applied depending on the amounts of the residual stresses in the plate. In their study, theoretical and finite element models were used and they gave favour to the obtained results of the finite element models. They concluded that the conditions of crack opening depend on the crack length and the amount of the applied loads and that the behaviour of the crack opening were similar in both elastic-plastic cases. Vamsi (2016) showed in his thesis that in plates with a hole, the increase of the size of the hole can led to limitations in using mathematical models for the analysis [13]. Kern et., al. (2016) explained the microstructural behaviour of solid and microcellular-voided composites of polypropylene with wheat straw reinforcing fibers using FEA modelling, mechanical test results and microscopy [14]. In plates with holes were made of composite materials, the angle of fiber inclination affects the deformations and stresses. Saddique and Mirzana (2016) showed that fiber at 30° can reduce the stress concentration factor while the fibers at 0° and 90° are considered to provide better performance regarding the stress and the deformation respectively [15].

In this research, a plate with a hole having four cracks distributed around the hole at 45° was investigated. Crack lengths of 5, 10, 15, 20, and 25 mm were used. Finite element models were utilized for this work.

2. Dimensional characteristics of the plate with a hole:

The plate chosen for the study was rectangular with a hole at the middle of the plate, the material was steel with a modulus of elasticity of 200 GPa. The



width, height and thickness of the plate were 150 mm, 300 mm and 1 mm respectively. The diameter of the hole was 100 mm. four cracks were added to the plate. These cracks were placed around the hole at 45° at each side. Crack 1 positioned to the top left of the hole at 45° while crack 2 positioned at 45° to the top right of the hole. Cracks 3 and 4 were positioned to the bottom right and bottom left of the hole respectively at an angle of 45° for each crack, and the type of crack is (EPFM). Figure 1 shows the model of the plate with a hole. Table 1 lists dimensions and cracks length for the plate models.

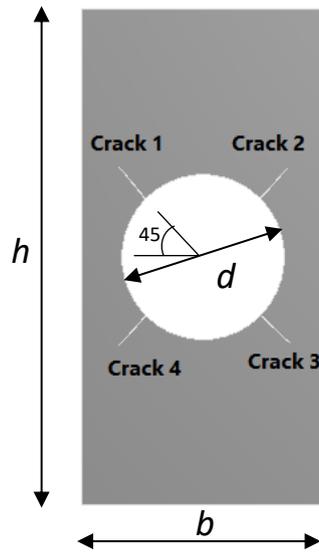


Figure (1): The plate with a hole showing the crack locations.

Table (1): Dimensions, loading and crack lengths of the plate models.

Model	Width(b)×Height (h)×Thickness (t)	Diameter (d) mm	Pressure MPa	Crack length L mm
1	150 ×300 ×1	100	150	5 mm
2	150 ×300 ×1	100	150	10 mm
3	150 ×300 ×1	100	150	15 mm
4	150 ×300 ×1	100	150	20 mm
5	150 ×300 ×1	100	150	25 mm

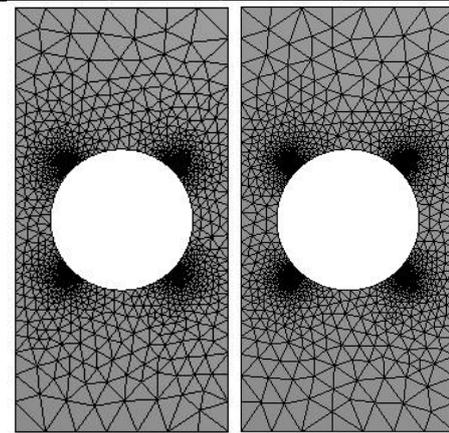
3. Finite element modelling

Finite element models were built for the plate with a hole. Five FE models with crack lengths of 5, 10, 15, 20 and 25 mm were proposed for this study. Each model had four cracks distributed around the hole at 45o at each side as shown in Figure 1. The element types were PLANE183 with 8-node- 2D and SURF 153-2D, the material was steel with modulus of elasticity of 200 GPa. The 3D geometry of the plate with cracks and holes is meshed and refined the contact region to obtain more accurate results. The mesh type used in this research is free mesh because it can give more accurate results when compared with the rectangular type. In Fig. 3, the mesh type and more dense area of the mesh are illustrated around the crack, see ([16-18]). The meshing method was chosen to be triangular. The finer mesh was applied along the crack edges. The number of divisions along the crack edge for crack lengths of 5, 10, 15, 20 and 25 mm was 20,

40, 60, 80 and 100 respectively. The meshed FE models with different crack lengths are depicted in Figure 2 and Figure 3. The number of nodes and elements for each model is listed in Table 2.

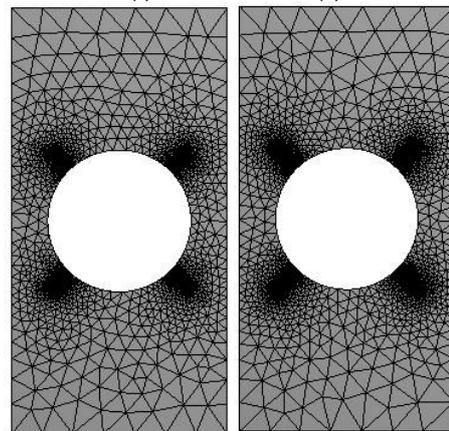
Table (2): Number of nodes and elements for the FE plate models

Crack length, mm	No. of nodes	No. of elements	Divisions along crack
5	8714	4178	20
10	12807	6147	40
15	16827	8081	60
20	21050	10114	80
25	25237	12131	100



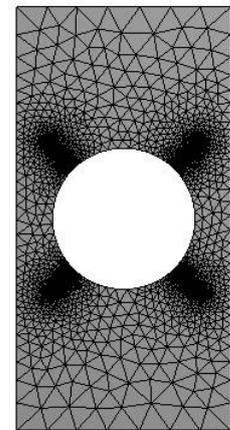
(a)

(b)



(c)

(d)



(e)

Figure (2): Meshing of the plate with a hole models with crack lengths of (a) 5 mm, (b) 10 mm, (c) 15 mm, (d) 20 mm and (e) 25 mm

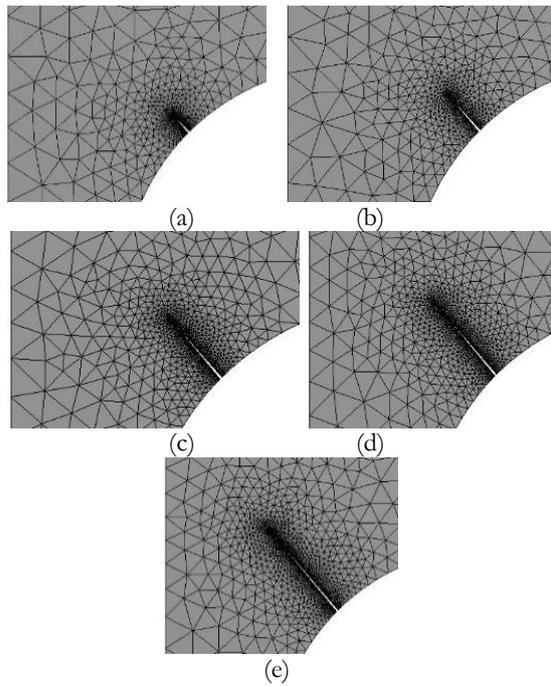


Figure (3): Finer meshing along the crack for models with crack lengths of (a) 5 mm, (b) 10 mm, (c) 15 mm, (d) 20 mm and (e) 25 mm

4. Loading and boundary conditions

A loading of tensile stress of 150 MPa was applied at the top side of the plate. The cantilever condition was adopted for the plate model where the bottom side of the plate was fixed, **cantilever** is a rigid structural element that extends horizontally and is supported at only one end. The loading condition shall be simulated using the ANSYS package. Static loading was applied to the models for 1 sec. The time of the simulation using ANSYS software where the stress was applied as a ramp signal through the interval was divided into 10-time steps to follow the graduate change in the resulted stresses and strains.

5. Results and discussions:

5.1 Effect of crack length on Von Mises stresses and strains

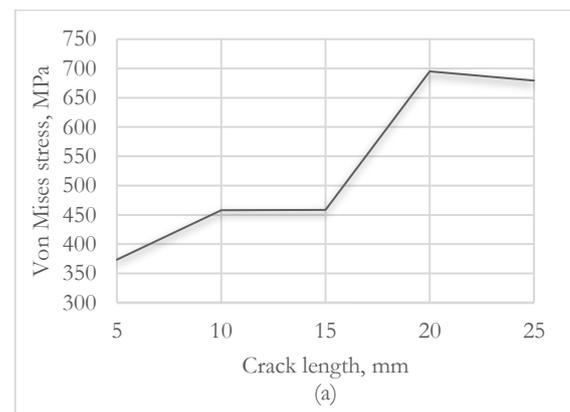
In this paper, five crack lengths were studied. The models were analyzed numerically using the ANSYS package. The resulted stresses and strains were collected at the top vortexes of each crack. Table 3 lists the values of the stresses and strains at the vortexes of cracks 1, 2, 3 and 4 (Figure 1) for the different crack lengths. Figure 4 shows the behaviour of the Von Mises stress against the crack length of each crack location, and it can conclude that the L=10 mm can be provided with uniform Von Mises distribution over the different crack positions.

The values of the stresses and strains were recorded at the tips of the cracks. For the crack lengths of 5, 10, 15, and 20 mm, the resulted in Von Mises stresses and strains were increased by increasing the crack length except for the crack length of 25 mm where the stresses and strains' controversial behaviour of a slight decrease in their values, and the optimum crack position for high capacity of Von Mises stresses and strains at crack position 3 as shown in figure 4.

This could be referred to as the behaviour of the stress intensity factors generated at the tips of the cracks as the shape of the cracks may have been changed and affected by the different crack lengths. It seems that there is a limit for the behaviour of the stresses against the crack lengths where this relationship changes when the crack length exceeds a certain size.

Table (3): Stress, strain and SIF values at the vortex of cracks 1, 2, 3 and 4 for each crack length

Crack length, mm	Crack 1		
	Stress, MPa	Strain	SIF MPa
5	373.48	1.64E-03	333.4
10	457.75	2.11E-03	506.64
15	458.26	2.11E-03	495.07
20	695.02	3.11E-03	692.95
25	679.33	3.01E-03	665.14
Crack 2			
5	401.38	1.75E-03	288.94
10	467.95	2.17E-03	522.08
15	490.12	2.20E-03	494.72
20	627.93	2.79E-03	639.99
25	587.43	2.61E-03	583.24
Crack 3			
5	358.51	1.59E-03	260.02
10	489.67	2.17E-03	476.2
15	568.45	2.51E-03	554.12
20	783.25	3.41E-03	734.95
25	683.35	2.97E-03	655.03
Crack 4			
5	333.57	1.52E-03	283.38
10	457.25	2.12E-03	503.79
15	461.27	2.17E-03	537.98
20	569.76	2.58E-03	594.19
25	566.86	2.52E-03	571.95



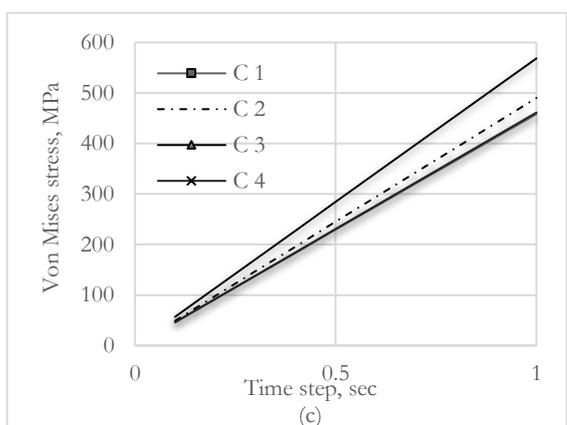
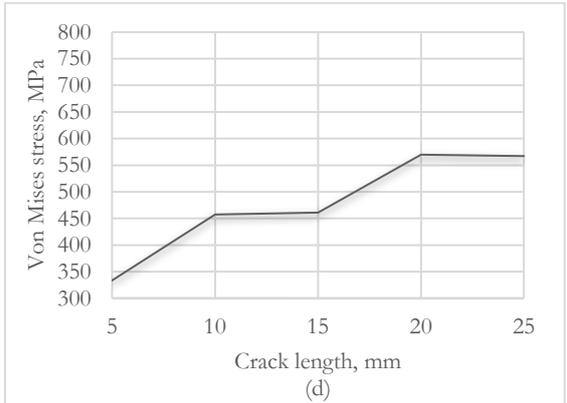
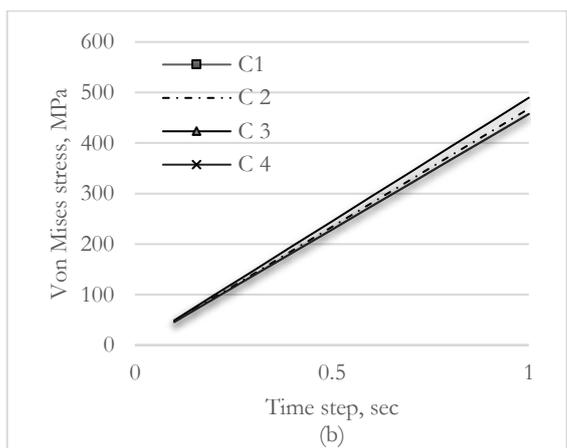
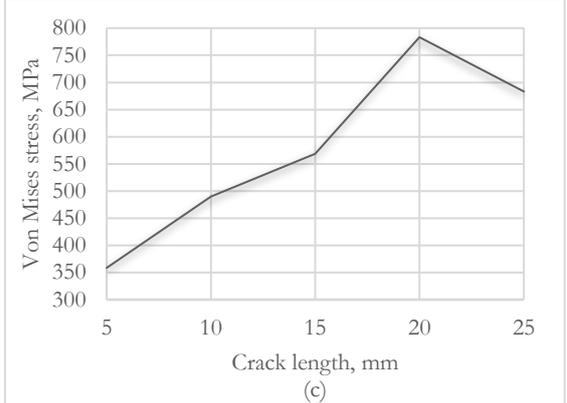
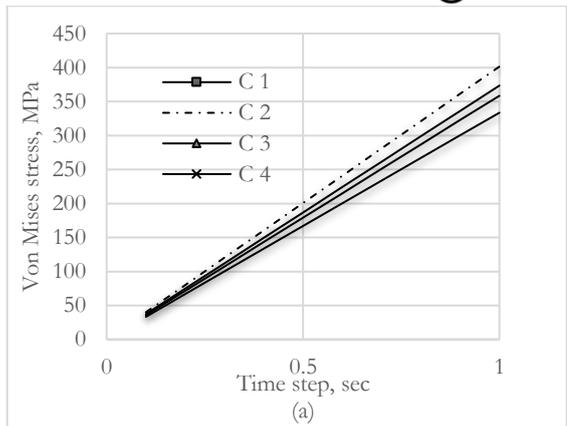
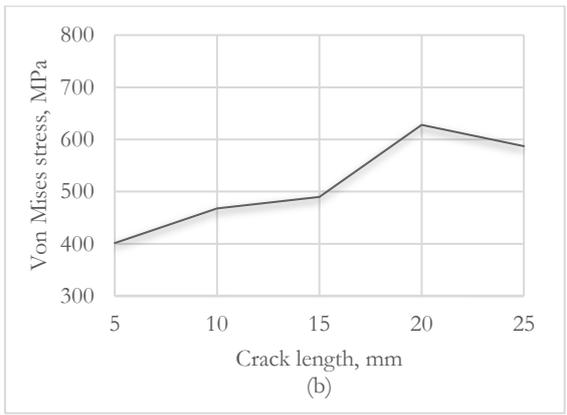
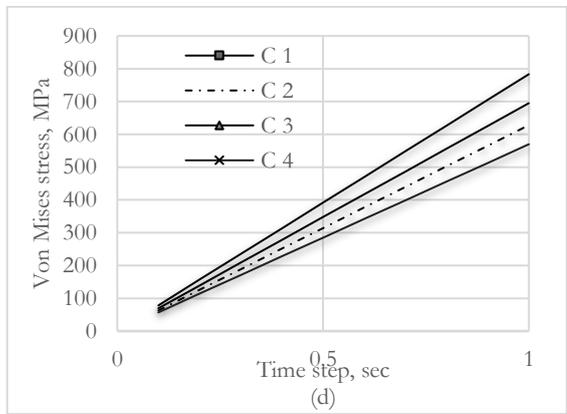


Figure (4): Effect of crack length (L) on Von Mises stresses at the vortex of (a) crack 1, (b) crack 2, (c) crack 3 and (d) crack 4

5.2 Effect of crack position on Von Mises stresses and strains

For each model with a specific crack length, the stresses were plotted for each crack position as each model had four crack positions as shown in Figure 1. Figure 5 shows the comparison of the collected stresses against each crack position for the different crack lengths.



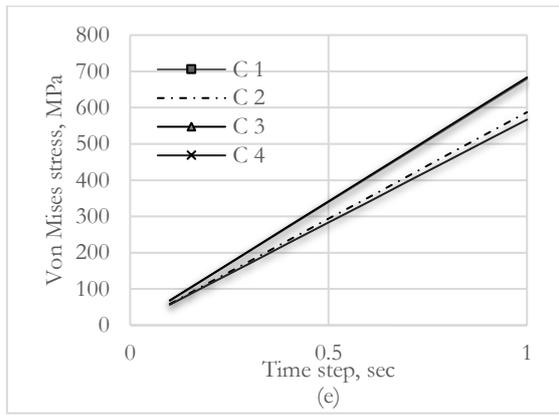


Figure (5): Comparison of Von Mises stresses at the vortex of cracks at position crack 1, crack 2, crack 3 and crack 4 with a crack length of (a) 5 mm, (b) 10 mm, (c) 15 mm, (d) 20 mm and (e) 25 mm.

Where the line with square marker represents crack 1, the dashed line represents crack 2, the line with triangle marker represents crack 3 and the line with cross marker represents crack 4

Figure 6 shows the finite element simulation comparison results of Von Mises stress distribution on the plate with both a hole and four cracks. The stress simulation was recorded during the tensile pressure loading at the top side of the plate while maintaining the bottom side of the plate fixed.

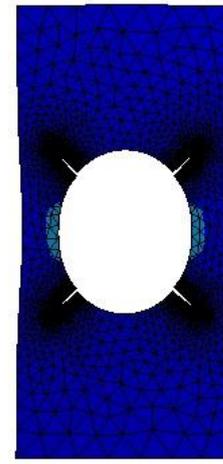
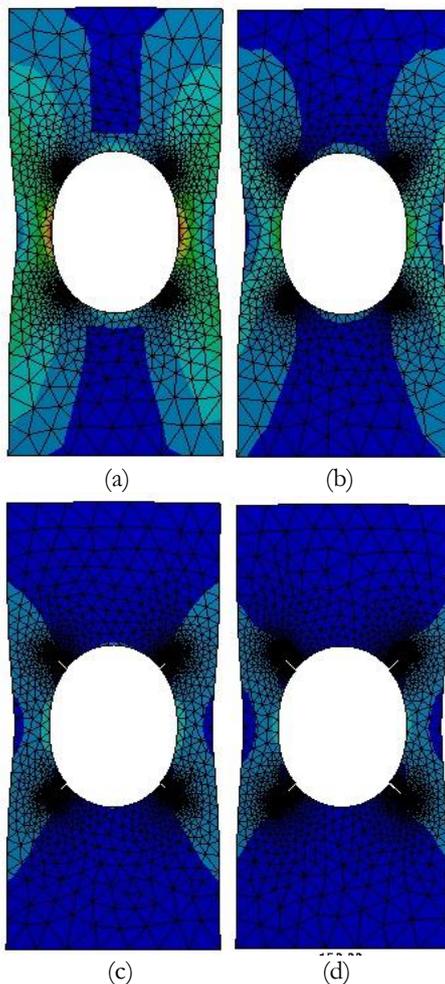


Figure (6): Stress distributions around the plate containing a hole surrounded by four cracks with crack lengths of (a) 5 mm, (b) 10 mm, (c) 15 mm, (d) 20 mm and (e) 25 mm

6. Conclusions:

For most of the four cracks distributed around the hole, increasing the crack length has led to an increasing both the Von Mises stresses and strains.

The effect of crack position on the resulted stresses was investigated at different crack lengths (5, 10, 15, 20, 25 mm). The results were not consistent as it appears that the crack length can change the whole stress distribution around the plate especially since there were four cracks around the hole in this study. For the crack length of 5 mm, crack 2 recorded maximum stress then crack 3 and then followed by crack 4. For crack lengths 10 mm and 15 mm, the same behaviour was observed where crack 3 recorded maximum stress then crack 2 followed by cracks 1 and 4 with similar results to cracks 1 and 4. For the crack length of 20 mm, crack 3 recorded maximum stress and then, crack 1 and then, crack 2 and then followed by crack 4. For the crack length of 25 mm, both cracks 1 and 3 recorded maximum stress, and then crack 2, followed by crack 4.

7. References:

- [1] K. D. Kumar, B. Ashish, and B. Vinod, "Evaluation of tensile properties of hybrid kevlar-glass reinforced epoxy composite for multi holes configuration," *Materials Today: Proceedings*, vol. 44, pp. 1065-1070, 2021.
- [2] G. Mekalke, M. Kavade, and S. Deshpande, "Analysis of a plate with a circular hole by FEM," *Journal of Mechanical and Civil Engineering*, pp. 25-30, 2012.
- [3] P. Gujar and K. Ladhane, "Bending analysis of simply supported and clamped circular plate," *International Journal of Civil Engineering*, vol. 2, no. 5, pp. 69-75, 2015.
- [4] S. Dhanjal and R. Arora, "Stress analysis of a rectangular plate with a circular hole using three-dimensional finite element model," *International Journal of Engineering, Business and Enterprise Applications (IJEBA)*, vol. 12, no. 1, pp. 77-80, 2015.



- [5] S. CHORFI and B. NECIB, "Crack propagation analysis around the holes in the plates under the effect of external stresses using the finite element model," 2016.
- [6] D. Jiang, Q.-H. Luo, W. Liu, and Y.-T. Zhou, "Thermoelectric field disturbed by two unequal cracks adjacent to a hole in thermoelectric materials," *Engineering Fracture Mechanics*, vol. 235, p. 107163, 2020.
- [7] O. L. Bowie, "Rectangular tensile sheet with symmetric edge cracks," 1964.
- [8] H. K. Kutter, "Stress analysis of a pressurized circular hole with radial cracks in an infinite elastic plate," *International Journal of Fracture Mechanics*, vol. 6, no. 3, pp. 233-247, 1970.
- [9] P. Weißgraeber, J. Felger, D. Geipel, and W. Becker, "Cracks at elliptical holes: stress intensity factor and finite fracture mechanics solution," *European Journal of Mechanics-A/Solids*, vol. 55, pp. 192-198, 2016.
- [10] W. Yi, Q.-h. Rao, S. Luo, Q.-q. Shen, and Z. Li, "A new integral equation method for calculating interacting stress intensity factor of multiple crack-hole problems," *Theoretical and Applied Fracture Mechanics*, vol. 107, p. 102535, 2020.
- [11] T. Chen, J. Yao, R. Liu, and L. Li, "Fatigue behaviour of steel plates with multi-holes repaired by CFRP," *Composite Structures*, vol. 242, p. 112163, 2020.
- [12] G. Wu, C. Aird, and M. Pavier, "The effect of residual stress on a centre-cracked plate under uniaxial loading," *International Journal of Fracture*, vol. 219, no. 1, pp. 101-121, 2019.
- [13] V. N. S. K. J. Parimi and V. Eluri, "Stress Analysis of Different Shaped Holes on a Packaging Material," ed, 2016.
- [14] W. T. Kern, W. Kim, A. Argento, E. C. Lee, and D. F. Mielewski, "Finite element analysis and microscopy of natural fibre composites containing microcellular voids," *Materials & Design*, vol. 106, pp. 285-294, 2016.
- [15] A. S. Shaik and I. M. Mirzana, "Stress concentration of rectangular plate with a hole made with composite material using finite element analysis," *IOSR J Mech Civil Eng*, vol. 13, no. 4, pp. 2278-1684, 2016.
- [16] Y.-K. Khdir, T. Kanit, F. Zaïri, and M. Naït-Abdelaziz, "A computational homogenization of random porous media: Effect of void shape and void content on the overall yield surface," *European Journal of Mechanics-A/Solids*, vol. 49, pp. 137-145, 2015.
- [17] Y.-K. Khdir, T. Kanit, F. Zaïri, and M. Naït-Abdelaziz, "Computational homogenization of plastic porous media with two populations of voids," *Materials Science and Engineering: A*, vol. 597, pp. 324-330, 2014.
- [18] Y. Khdir, T. Kanit, F. Zaïri, and M. Naït-Abdelaziz, "Computational homogenization of elastic-plastic composites," *International journal of solids and structures*, vol. 50, no. 18, pp. 2829-2835, 2013.