Estimating Transfer Function of Below-Knee Prosthesis at Two Phases of Gait Cycle

Mahmud Rasheed Ismail
Prosthetic and Orthotic Eng. Dept.
Al-Nahrain University
mahmech2001@yahoo.com

Imad Zuhair Ghani
Mechanical Eng. Dep.
Al-Nahrain University

Abstract:

The modern development in prosthetics field demand the evaluation of the dynamical behavior and automatic control .The key process in the design and implement of these devices is the determination of the model parameters inherited with the transfer function .In such complicated structures it is so difficult to evaluate transfer function analytically ,however experimental approaches can serve as a simple and effective tool for estimating transfer function and model parameters .In this regard computer software such as Matlab is used .System Identification SID refers to the method for estimating the system transfer function from experimental tests by using computer .In the present paper; SID method is employed for analyzing below-knee prosthesis leg .In order to simulate with the practical requirement for design and evaluation ,two phases of human gait are considered ,namely; swing phase and single support of stance phase .The validity of this method is firstly checked by applying it on clamped-clamped beam model where the required parameters are evaluated and compared theoretically (via modal analysis) and experimentally (via System identification) .It is found that ; the error in estimating the transfer function parameter of beam is not exceeded 6% . Then the transfer function of the prosthesis are estimated for two phases of gait cycle .It is found that; the estimated transfer function of the prosthesis leg is highly affected by the phase type of gait cycle , where ;the natural frequency highly increases, the static gain decrease for support phase as compared with the swing phase ,however the damping ratio does not affected .

Keywords: Transfer function, Modal analysis, Below-knee prosthesis, Gait cycle, FFT

1. Introduction

Investigating the dynamical behavior of complicated structures from analyzing experimental data took wide attention for researchers and engineers .The main goal of this analysis is to evaluate the system parameters associated with the resulting curve relating the input and output data . In this method the system is subjected to an input excitation to produce an output response .Both input and output signal waves are sampled at equal time interval to generate set of data .The data is then transformed to frequency domain by Fast Fourier Transformation FFT and proceed in computer to find the optimum transfer function. By evaluating the transfer function the dynamical parameter of system such as, natural frequency, damping and gain can be evaluated. A wide information about the vibration, stability and control characteristics can be clearly investigated for the target system. System Identification SID is a computer software used for performing such an analysis, as example in Matlab. This field of research has wide practical applications in engineering such as fault detection in automatic machines Basseville et.al. [1], damage detection Mevel, et.al. [2], health monitoring Zimmerman et.al. [3], car body, Sabri [4] airplane, Goethals and Moor[5], automotive and aerospace applications, Bart Peeters et al.[6] mobility analyzer (DMA) in the nanometer size range Hummes,et al.[7] ducted laminar premixed flame Karimi, et.al.[8] and vehicle component testing Peeters et.al[9]. In simple or regular systems and structures the transfer function can be evaluated in straight forward analysis, however for complicated systems it is not so because of the complication of stiffness and mass as well as the existed of nonlinearity .In such systems the theoretical analysis becomes tedious .In this regard; finite element and state space methods are normally used Ewins [10]. The accuracy obtained in these methods depend on the number of elements used which are normally large Heylen et. al.[11] The main challenge of determination of the system parameters is how to fit the complex transfer function curve to the standard forms of transfer function with certain confidence.

Cauberghe .[12] Researchers attempted many numerical and statistical methods to improve the optimization (minimizing error and time labor) process ;for example Guillaume et.al.[13] used frequency-domain maximum likelihood , Van Der et.al.[14] employed fast-stabilizing frequency domain method , Guillaume et.al.[15] introduced a poly-reference implementation of the least-squares and Canales and Mevel [16] applied the quasi newton based system Due to the recent developments in the field of prosthetic which involve the robotic and intelligent devices such as; the C-leg , feeling leg and robotic prosthesis hand the demand for evaluating the system parameter becomes essential for design and implement.

In this paper, the estimation of the transfer function is attempted for the below-knee prosthesis leg as an example of complicated object which consists of many elements such as ;plastic socket ,steel rod ,plastic foot and many connection joints.In order to simulate with the practical requirements ,two phases of human gait cycle will be investigated ,namely the swing phase and single support of stance phase .For the validation purpose a classical beam with clamped-clamped boundary condition is used in which the system parameter are evaluated in both theoretical and experimental methods

2. Theoretical Consideration

In order to check the validity of the present method a typical beam model will be investigated to evaluate the system parameters theoretically by using Modal Analysis method .The beam model is shown in figure 1.

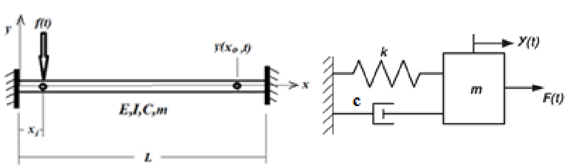


Figure 1: The beam System and equivalent m-c-k system

The equation of force vibration of damped beam according to Euler-Bernoulli theory can be written as [17];

$$EI\frac{\partial^4 y}{dx^4} + C\frac{\partial y}{dt} + \rho A\frac{\partial^2 y}{dt^2} = f(t)\delta(x - x_j) \qquad (1)$$

Where the concentrated force is located at x_j.

For Modal Analysis one can seek the following solution ;

$$y(x,t) = \sum_{s=0}^{N} \phi_s(x) q_s(t) \qquad (2)$$

Where $\phi_s(x)$ and $q_s(t)$ stand for mode shape and generalize coordinates .Substituting the solution (2) into the equation (1) results ;

$$EI\sum \phi_s^{IV} q_s + C\sum \phi_s \dot{q}_s + \rho A\sum \phi_s \ddot{q}_s = .\, f(t)\,\delta(x - x_j) \qquad (3)$$

Where $\phi_s(x)$ and q(t) are replaced by q_s and ϕ_s for simplicity.

Now, multiplying Eq. (3) by the boundary residual series $\phi_r(x) = \sum_{r=1}^{N} \phi_r(x)$

and integrating over the beam length (0 to L) , the following matrix equation can be obtained [18];

$$[K]\{q\} + [C]\{\dot{q}\} + [M]\{\ddot{q}\} = f(t)\varphi(xj) \qquad (4)$$

The matrices [K] and [M] are diagonal due to the orthogonally property of the normal modes that is;

$$\int_0^L \phi_s(x)\phi_r(x)dx = \begin{cases} 1\, for...s = r \\ 0\, for....s \neq r \end{cases} \quad \text{and}$$

$$\int_0^L \phi_r^{IV}(x)\phi_s(x)dx = \begin{cases} \lambda_s^4\, for...s = r \\ 0\, for...s \neq r \end{cases} \qquad (5)$$

The elements of the stiffness and mass matrices are as the follows ;

$$K_{s,s} = EI\int_0^1 \phi_s^{IV}(x)\phi_s(x)dx \qquad (6)$$

$$m_{s,s} = \rho A\int_0^1 \phi_s(x)^2\, dx \qquad (7)$$

To evaluate the transfer function the harmonic analysis must be considered , assuming harmonic motion and harmonic force one can write ;

$$q_s(t) = Q_s e^{i\omega_s t} \text{ and}$$
$$f(t) = Fe^{i\omega t} \qquad (8)$$

Substituting equations (8) into equation (4) and making the use of transfer function definition the following equation is resulted ;

$$H_s(\omega) = \frac{Q_s(\omega)}{F_s(\omega)} = \frac{K_S\varphi_S(xj)}{[1+2\zeta\frac{\omega}{\omega_S}+\frac{\omega^2}{\omega_S^2}]} \qquad (9)$$

When $s=1$,the equivalent single degree of m-c-k system can be obtained,hence the corresponding transfer function becomes ;

$$H_1(\omega) = \frac{K_S\varphi_1(xj)}{[1+2\zeta\frac{\omega}{\omega_1}+\frac{\omega^2}{\omega_1^2}]} \qquad (10)$$

Although the above analysis is general for any beam boundary condition, in this paper, a focus is made on clamped-clamped boundary conditions In this case the shape function $\phi_s(x)$ is [17] ;

$$\phi_s(x) = \sin\lambda_s x + \sinh\lambda_s x - \frac{\sinh\lambda_s - \sin\lambda_s}{\cosh\lambda_s - \cos\lambda_{s\ s}}(\cos\lambda_s x + \cosh\lambda_s x) \qquad (11)$$

Where the eigenvalues λ_s are the roots of the characteristics equation $\cosh\lambda_s L \cos\lambda_s L = 1$
The natural frequency can be obtained as;

$$\omega_s = (\lambda_s L)^2\sqrt{\frac{EI}{ML^4}} \qquad (12)$$

Equations 10 together with equations 6,7 , 11 and 12 can give the theoretical transfer function and system parameters of the clamped-clamped beam.
Due to the difficulty of evaluating the damping theoretically .The damping was measured experimentally by using the concept of the logarithmic decrement of the transient response curve which is available from hummer test .

3. Experimental Estimation

When a set of experimental input data X(t) and output data Y(t) are available ,the transfer function H(ω) can be obtained from the following relation [12]

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{FT\ of\ output}{FT\ of\ input} \qquad (13)$$

Equation 13 leads to complex transfer function with magnitude and phase shift which can written as ;

$$H(\omega) = A(w)e^{i\theta(\omega)} \qquad (14)$$

Now if equation 14 is plotted against a range of frequencies and fitted to optimal bode plot it is possible to estimate the TF in terms of the system parameters (gain, natural frequencies, damping …etc)

The method of auto-regression can be used which leads to a classical linear least squares .In order to reduce the fitting error the following cost function J(b) is used [10];

$$J(b) = \int_0^\infty \left|G(i\omega,b) - A(w)e^{i\theta(\omega)}\right|^2 d\omega \qquad (15)$$

Where $G(i\omega,b)$ denotes the desired transfer function of fitting .For desecrate data this integration is replaced by summation;

$$J(b) = \frac{1}{2}\sum_{n=1}^{N}\left|G(i\omega,b) - A(w)e^{i\theta(\omega)}\right|^2 \Delta\omega \qquad (16)$$

The system transfer function is assumed to have a linear structure expressed in factored form. This is the general form familiar to Bode plot which can be represented in the following factored form [10];

$$G(s,b) = \prod_{j=1}^{M} g_m(s,b)_j \qquad (17)$$

Where g_m can take any of the following forms;

$$s, (1+\tau s), (1+2\zeta s+\omega^2 s^2), \frac{1}{(1+2\zeta s+\omega^2 s^2)} \ldots\ldots etc \qquad (18)$$

In Matlab software the so called “System Identification “ is available to work with measuring data or observations .This includes FFT transformation, complex frequency evaluation , data correlation and outo correlation ,optimization ,curve fitting to bode plot and system parameters estimation

4. Experimental Work

Three experiments are carried out to evaluate the transfer function between two selective points. One test is for clamped- clamped beam (figure 2) and two for prosthesis leg at swing and support phases of human gait cycle (figures 3 and 4) .

The beam model sample is Aluminum with circular cross section which has the following specifications (L=0.52 m, d=10 mm ,E=71x10^9 N/m^2 and ρ = 7800 kg/m^3) .

The below-knee prosthesis is made from three major parts; the socket which is made from Perlon (623T3) with Carbon fiber stockinet (with layers 4 p-2c-4 p)

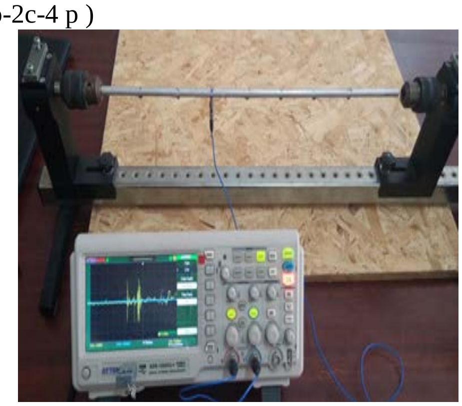


Figure 2: Beam test


Figure 3: Prosthesis leg at swing phase test


Figure 4: Prosthesis leg at support phase test

The steel rod and the plastic foot .To introduce the swinging motion two pines are attached on steel clamper ring fitted on the socket .The pines then hinged with another U shape link by means of two ball bearing, finally the prosthesis is over hanged on a cantilever beam as shown in figure 3.For single support phase the foot of the prosthesis is trapped between two beam channels to constrain the motion as shown in figure 4.

In all tests the impact hummer is used to introduce an impulse force at the input point .The output response was picked out by using the accelerometer . A charge amplifier was employed to amplify the response. The input and output signals were displayed on the oscilloscope .The input and output were sampled and then exported and saved in the laptop .Figure 5 presents a block diagram for the measuring equipment used in the prosthesis leg tests .The tests are repeated for several points in forward and reverse manners (i.e: exchanging the input and output points).

Figure 6 is a photograph picture of the real input and output signals as observed in the oscilloscope for one of the beam test .The stored fiels are then exported to Matlab worksheet for estimating the transfer functions .

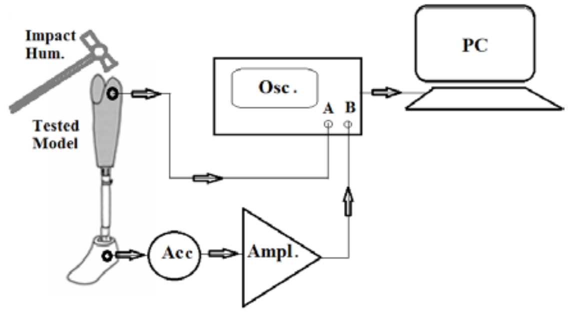


Figure 5: Block diagram of measuring equipment used in Prosthetics leg tests

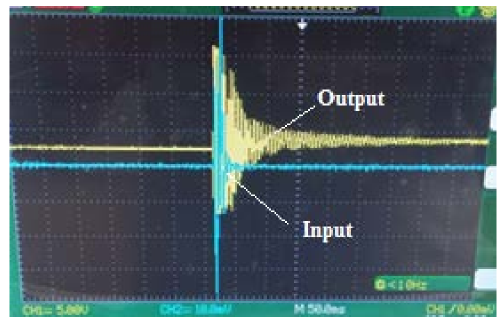


Figure 6: Real input and output wave signals for beam

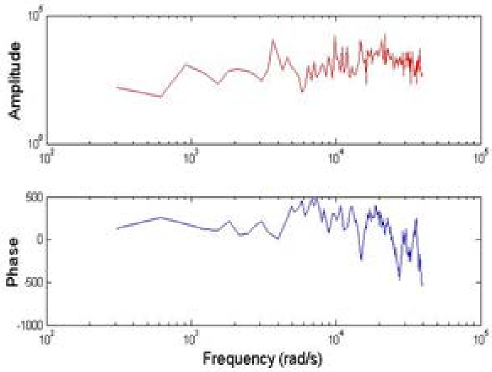
5. Results and Discussions

The experimental results for estimating the System parameters of the clamped-clamped beam by using system identification obtained in Matlab are shown in figures 7-a to d .In figure 7-a the input and output data are plotted in time domain .The FFT of data is performed and plotted in figure 7-b . Figure 8-c shows the autocorrelation and cross correlation of data exported for best fitting to the slandered 2nd transfer function .Finally figure 9-d presents the bode plot of the transfer function. The estimated transfer function for the beam which is obtained by Matlab as the follows ;

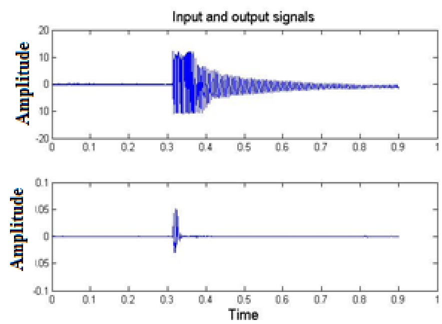
$$H(\omega) = \frac{70.304}{[1+2x0.0078\frac{\omega}{1000}+\frac{\omega^2}{1000^2}]} \quad (19)$$

The natural frequency and static gain are calculated by using equations 6 and 7, and the damping ratio was measured experimentally.

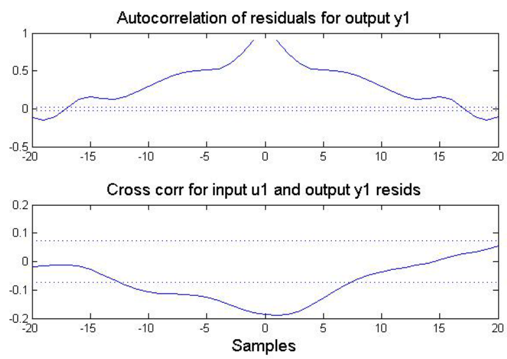
In table 1, the experimental SID and theoretical results are collected and the percentage error are calculated .This table show a good agreement between the two methods where the maximum error is not exceeded 6% .This prove the validity of using the estimation method with confidence .



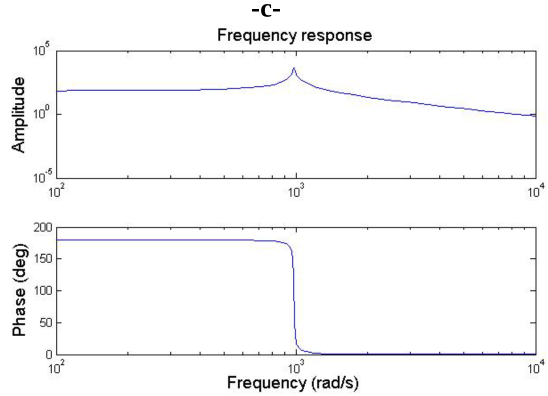
-a-



-b-



-c-



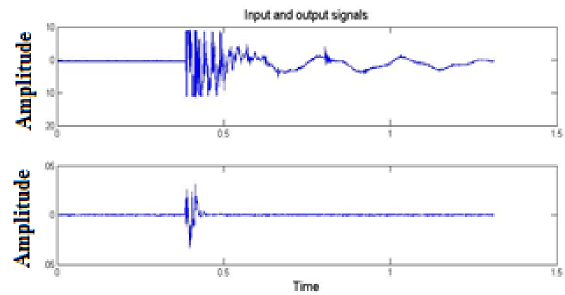
-d-

Figure 7: SID plots of clamped-clamped beam

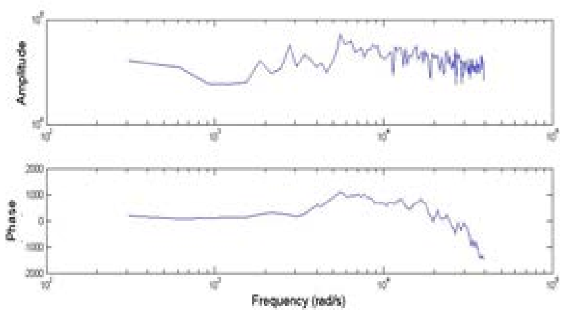
Table 1: Theoretical and experimental parameters of clamped-clamped beam

System parameters	Theoretical	System Ident.	Error %
Natural frequency (r/s)	987	1000	1.8
Damping ratio	0.0083	0.0078	6%
Gain (m/N)	69.5	70.304	1.15

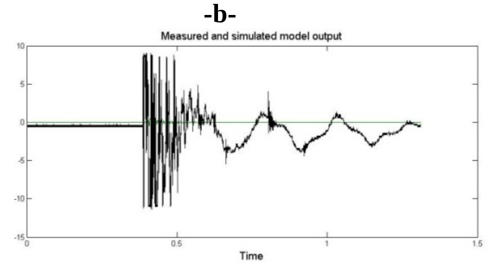
Figures 8-a to f; present the following plots of the below knee prosthesis at swing phase ; input and output data in time domain ,input and output data in frequency domain ,measured and simulated system output ,data correlation ,bode plot and the step response ,respectively . For the same prosthesis but at support phase the same sequence of plots of SID are shown in figures 9-a to f .



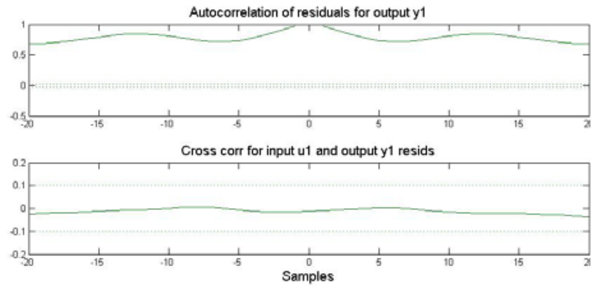
-a-



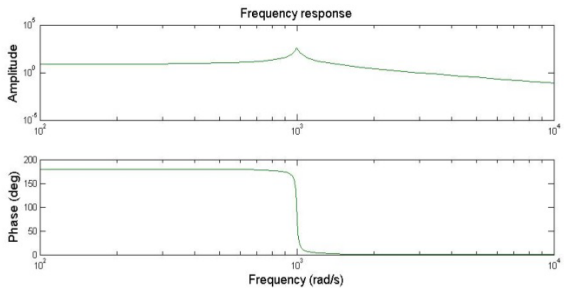
-b-



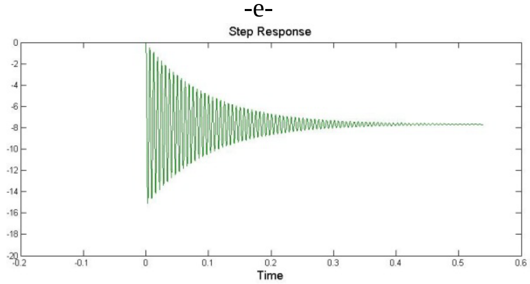
-c-



-d-

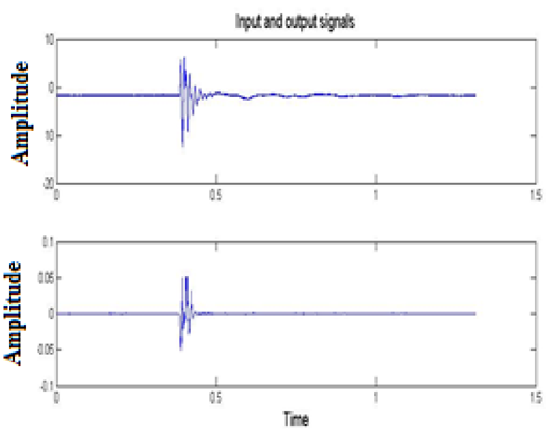


-e-

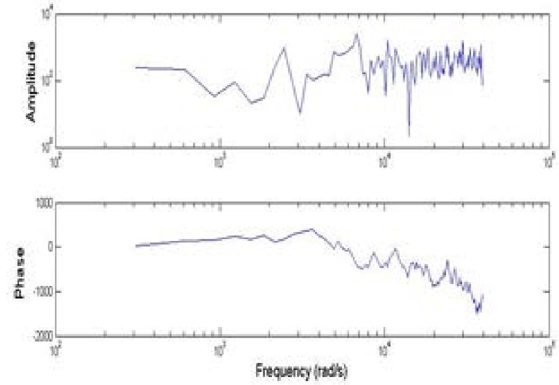


-f-

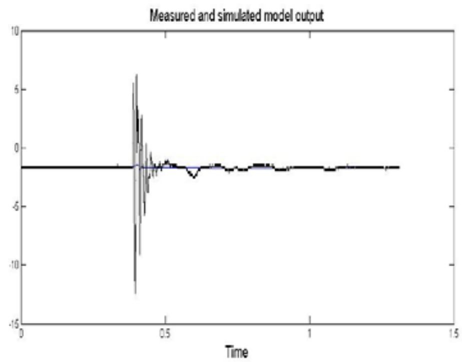
Figure 8: SID plots of prosthesis at swing phase

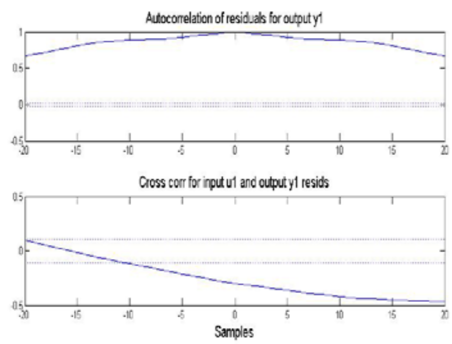


-a-

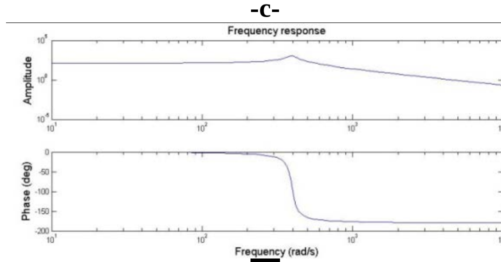


-b-

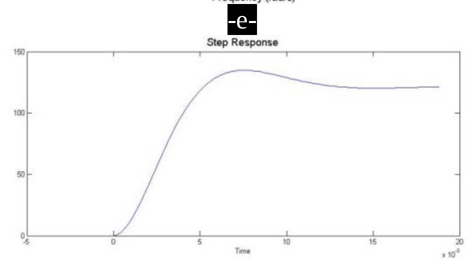




-c-



-e-



-f-

Figure 9: SID plots of Prosthesis at support phase

The estimated transfer function of the below knee prosthesis at swing and support phase which are obtained from Matlab are as the follows ;

$$H(\omega) = \frac{2.677}{[1+2x0.0103\frac{\omega}{892.6}+\frac{\omega^2}{892.6}]} \text{ for swing phase} \quad (20)$$

$$H(\omega) = \frac{121.73}{[1+2x0.011\frac{\omega}{512.82}+\frac{\omega^2}{512.82^2}]} \text{ for support phase} \quad (21)$$

Comparing equation 20 and 21 indicates the large different in the transfer function parameters between the two phases of the gait cycle. This can attributed to the fact that at support phase the prosthesis device is subjected to more deformation than the swing phase .This deformation comes from the effect of the ground reaction force GRF .Accordingly , the stiffness is highly decreased and hence the natural frequency decrease.

6. Conclusions

From the discussions of the results the following conclusions can be derived;

1- The validity of the present method is checked ;it is found that the percentage error in the transfer function parameters between the theory and experiments is not exceeded (6 %) .

2- The transfer function of the prosthesis leg is affected by the phase type of gait cycle ,it is found that ;at the swing phase ;the natural frequency increase, the static gain decrease and the damping is not affected

7. List of Symbols

A	Beam cross sectional area m^2
C	Damping coefficient NS/m
E	Modulus of Elasticity N/m^2
$H_s(\omega)$	Transfer function
J(b)	Cost Function
q(t)	Time function
ϕ	Space function
ρ	Density kg/m^3
ω	Natural Frequency r/s
δ	Dirac delta function
λ	Characteristics root of beam frequency equation
τ	Time delay ,sec

8. References

[1] M. Basseville, M. Abdelghani, and A. Benveniste. Subspace-Based," Fault Detection Algorithms for Vibration Monitoring. Automatica", 36(1):101–109, January 2000.

[2] L. Mevel, and M. Goursat. M. Basseville, ,"Statistical System-Based Damage Detection and Localization :Subspace-Based Residuals and Damage-To-Noise Sensitivity Ratios." Journal of Sound and Vibration, 275(3):769–794, August 2004.

[3] D. C. Zimmerman, M. Kaouk, and T. W. Simmermacher. ,"Structural Health Monitoring Using Vibration Measurements and Engineering Insight", Journal of Mechanical Design(1990), 117(B):214–221, 1995.17th IFAC World Congress (IFAC'08) Seoul, Korea, July 6-11, 2008

[4] Sabri Sentürk ,"Experimental Determination of Transfer Functions For a Car Body-In-White,",A thesis submitted to the graduate school of natural and applied sciences of the SIDdle east technical university,2004

[5] I.Goethals. and B. De Moor. "Subspace Identification Combined With New Mode Selection Techniques for Modal Analysis of an Airplane". In Proceedings of the 13th IFAC Symposium on System Identification (SYSID 2003), Rotterdam, The Netherlands, September 2003

[6] Bart Peeters, Patrick Guillaume, Herman Van der Auweraer,Bart Cauberghe, Peter Verboven, Jan Leuridan, "Automotive and Aerospace Applications of The Polymax Modal Parameter Estimation Method"The International Modal Analysis Conference, Kissimmee (FL), USA, February 2003.

[7] Betlef Hummes ,Frank stratmann ,Heinze Fissan "Experimental Determination of Transfer Function of a Differential Mobility Analyzer (DMA) in The Nanometer Size Range ,1999.

[8] N. Karimi, S. H. Jin and M. J. Brear ,"An Experimental Study of The Transfer Function of a Ducted, Laminar Premixed Flame", 16th Australasian Fluid Mechanics Conference Crown Plaza, Gold Coast, Australia 2-7 December 2007,1340-1343.

[9] B.Peeters, P. Van Der Linden and C. De Veuster. ,"Performance of Miniature Shakers for Vehicle Component Testing." In Proceedings of IMAC 22, the International Modal Analysis Conference, Dearborn (MI), USA, January 2004.

[10] D. J. Ewins. ."Modal Testing: Theory, Practice and Application.", Research Studies Press Ltd, Hertfordshire, 2000.

[11] W. Heylen, S. Lammens, and P. Sas. "Modal Analysis: Theory and Testing. ",Department of Mechanical Engineering,Katholieke Universiteit Leuven, Leuven, Belgium,1998.

[12] B. Cauberghe. "Applied Frequency-Domain System Identification in the Field of Experimental and Operational Modal Analysis". PhD thesis, Vrije Universiteit Brussel, May 2004.

[13] P.Guillaume, P. Verboven and S. Vanlanduit. "Frequency-Domain Maximum Likelihood Identification of Modal Parameters With Confidence Intervals". In Proceedings of ISMA 23, the International Conference on Noise and Vibration Engineering, Leuven, Belgium, 16–18 September 1998.

[14] H.Van Der Auweraer, P. Guillaume, P. Verboven And S. Vanlanduit.," Application of a Fast-Stabilizing Frequency Domain Parameter Estimation Method". ASME Journal of Dynamic Systems, Measurement, and Control, 123(4), 651–658, 2001.

[15] P.Guillaume, P. Verboven, S. Vanlanduit, H. Van Der Auweraer And B. Peeters. ,"A Poly-Reference Implementation of The Least-Squares Complex Frequency-Domain Estimator". The International Modal Analysis Conference, Kissimmee (FL), USA, February 2003.

[16] G. Canales and L. Mevel. ."A Quasi Newton Based System Validation Procedure for Improving Parameter Estimation",.In Proceedings of the (IOMAC), Aalborg, DK,April 2007

[17] P.Verboven."Frequency Domain System Identification for Modal Analysis." PhD Thesis, Vrije Universiteit Brussel, Belgium, 2002

[18] R.Pintelon and J. Schoukens..' System Identification: a Frequency Domain Approach". IEEE Press, New York, USA, 2001

تخمين دالة الخصائص لطرف صناعية تحت الركبة ولطورين من المشية

محمود رشيد اسماعيل
قسم هندسة الاطراف والمساند الصناعية
جامعة النهرين

عماد زهير غني
قسم الهندسة الميكانيكية
جامعة النهرين

الخلاصة

التطويرات الحديثة في مجال الأطراف الصناعية تتطلب تقييم السلوك الديناميكي و عمليات السيطرة.الذاتية , ان اهم عملية في تصميم وتنفيذ هذه الأجهزة هو تحديد معالم النموذج المرتبطة مع دالة الخصائصTF .في مثل هذه الاجسام المعقدة من الصعب جدا ايجاد دالة الخصائص بطريقة تحليليه، ولكن النهج التجريبية يمكن أن توفر أداة بسيطة وفعالة لتقديرهما عبر استخدام تقنية تعريف الموديل .في هذا المجال يتم استخدام برنامج الكمبيوتر مثل برنامج الماتلاب. في هذا البحث تم توضيف الطريقة لتحليل طرف صناعي تحت الركبة. ومن أجل المحاكاة مع المتطلبات العملية تم اختيار مرحلتين من مشية الإنسان، وهما طور التارجح وطور الاسناد. تم في البداية عمل فحص لهذه الطريقه من خلال تطبيقها على عتبة محكمة الاسناد لايجاد الخصائص المطلوبة ومقارنتها من الناحية النظرية (عبر تحليل الاطوار) وتجريبيا (عن طريق تحديد نموذج) حيث تبين ان نسبة الخطأ لا تتعدى6% لحالة العتبة ,ومن ثم تم تطبيق الطريقة التجريبية على الطرف الصناعي ولطورين من دورة المشي. حيث وجد ان الدوال المخمنة للطرف الصناعي تتاثر بشدة بنوع طور المشية حيث ان التردد الطبيعي يزداد بشدة والربح يقل لحالة طور التارجح بالمقارنة مع طور الاسناد في حين ان نسبة التخميد لم تتاثر.