# Investigating of A Modified Model for Human Eye Movement 

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#### Abstract

In this research a biological model was modified upon a homomorphic sccadic eye movement model by adding and calculating the mass of eye and the elasticity of eye tendon and take a primary position only. According to this modification, equation with six order was appeared which was processed with a MATLAB SIMULINK to get a desire results of theta, velocity and acceleration in the normal case. For the same model, a disease was estimated to evaluate the results of theta and how it is affected by this disease which was estimate the increase and decrease happened in the primary elastic element and viscosity. After theta controller was added to the model to control the system and get the desire angle which is equal to $15^{\circ}$ in the normal case while in the system with disease, the PI controller that used can control only the system during normal case while in the system with disease, the PI controller could not control the system to get the desired angle.


## Introduction

Today science is witnessed major developments and rapid increase in recent creations and invitations in all fields including medical engineering, which is witnessed a marked improvement in human creations that serve this field and explain the design of the mechanical movement of the human eye, which consists of three movements, primary and secondary and tertiary.
Biological systems are becoming more appealing to approaches that are commonly used in systems theory and suggest new design principles that may have important practical applications in man-made systems. Modeling the eye plant in order to generate various eye movements has been the topic among neurologists, physiologists, and engineers for a long time. The eyes rotate with three degrees of freedom, i.e., horizontal, vertical and torsional. Addressing the question of modeling the eye plant to mimic the realistic eye movements involves a three dimensional approach. In this work, Hill type
musculo-tendon complex is used and the effects of extra-ocular pulleys are studied. The model proposed by Martin and Schovanec in 1999, for horizontal eye movement has been used as a basic starting point [1].
Rotations of the eye are generated by the torques that the eye muscles apply to the eye. The relationship between eye orientation and the direction of the torques generated by the extra-ocular muscles is therefore central to any understanding of the control of threedimensional eye movements of any type [2].

## Anatomy of Human Eyes

A schematic representation of the top view of a right human eye is shown in Fig.(1), the image shows the orbita containing the bulbus and the six extra ocular eye muscles.
Four of the extra ocular eye muscles are straight muscles:

- Medial rectus muscle,
- Lateral rectus muscle,
- Inferior rectus muscle,
- Superior rectus muscle.

Additionally, there exist two oblique muscles:

- Inferior oblique muscles,
- Superior oblique muscles.

The four straight muscles and the Superior oblique muscles originate in the annulus tendineus communis (or annulus of Zinn), a small tendon ring lying at the back of the orbita. This part of a muscle is called origin. The inferior oblique muscle originates in the front part of the orbita, near the nasal bone in the orbital wall. All extra ocular eye muscles are inserted into the bulbus and this part of an extraocular muscle (EOM) is called insertion. The pathway between the origin and the insertion of a muscle is called muscle path, whereas the part of the muscle path clinging to the bulbus is often referred to as arc of contact, where the muscle mostly consists of tendons. The specific position, where the muscle loses contact with the bulbus, is named point of tangency. [3]


Fig.(1) The top view of a right human eye [3]

## Diseases of the Muscles and Nerves

Diseases of the muscles and nerves include four relatively common conditions: [4]

## Strabismus

Strabismus is when the eyes are not straight. It usually starts in early childhood. The squint may be in-turning (esotropia) or out-turning (exotropia). If left untreated a squint in childhood can lead to disuse of the eye, with suppression of the visual stimuli leading to loss of vision known as amblyopia.

## Diplopia (Double Vision)

Ptosis: is an inability to open the eye normally.

## Lagophthalmos

Lagophthalmos is an inability to close the eye. It usually due to paralysis of the orbicularis oculi muscle following facial nerve paresis. It is an important sign and a complication of leprosy.

## The Rotations of Eye

The rotations an eye can perform around the different axes are the following:

- Adduction and abduction describe a rotation toward the nose and away from the nose around the z-axis, respectively.
- Elevation and depression specify an upward and downward rotation of the eye around the xaxis.
- Intorsion and extorsion define a rotation around the $y$-axis from the top of the bulbus toward the nose and away from it.
Fig.(2), depicts a graphical illustration of the different directions the left and the right eyes can rotate. [3]


Fig.(2) the six different directions an eye can rotate. [3]

## Modified Eye Modeling and Control

The mechanical components of the oculomotor plant is described in Fig.(3). The muscles are shown to be extended from equilibrium, a position of rest, at the primary position, consistent with physiological evidence. The average length of the rectus muscle at the primary position is approximately 40 mm , and at the equilibrium position it is approximately 37 mm . $\theta$ is the angle of the eyeball which is deviated from the primary position, variable $x$ is the length of arc traversed. When the eyes is at the primary position, both $\theta$ and $x$ are equal to zero. Variables $X_{6}-X_{7}$ are the displacements from equilibrium for the stiffness elements in each muscle, $\theta$ is the rotational displacement for passive orbital tissues. Values $X_{p 6}-X_{p 7}$ are the displacements from equilibrium for each of the variables $X_{6}-X_{7}$ at the primary position. The total extension of the muscle from equilibrium at the primary position is $X_{p 6}+X_{p 1}+X_{p 2}$ or $\quad X_{p 7}+X_{p 3}+X_{p 4}$, which equals approximately 3 mm . It is assumed that the lateral and medial rectus muscles are identical such that $x_{p 6}=x_{p 7}, \quad X_{p 1}=X_{p 3} \quad$ and $X_{p 2}=X_{p 4}$. The radius of the eyeball is $r$ [5].


Fig.(3) Diagram of mechanical components of oculomotor plant.

Fig.(3), illustrates the mechanical components of the oculomotor plant for horizontal eye movements, the lateral and medial rectus muscle and the eyeball. The agonist muscle is modeled as a parallel combination of viscosity $B_{2}$ and series elasticity $K_{s e}$, connected to the parallel combination of active-state tension generator $F_{a g}$, viscosity element $B_{1}$ and length-tension elastic element $K_{l t}$, for simplicity agonist viscosity is set equal to antagonist viscosity. The antagonist muscle is similarly modeled with a suitable change in active-state tension to $F_{\text {ant }}$. Each of the elements defined in the oculomotor plant is ideal and linear. The eyeball is modeled as a sphere with moment of inertia $J_{p}$ connected to a pair of viscoelastic elements connected in series. By summing the force acting at junctions 2 and 3 and the torque acting on the eye ball at junction 5, a set of six equations is written to describe the oculomotor plant:

$$
\begin{align*}
& F_{a g}=K_{l t} X_{2}+B_{1} \dot{X}_{2}+K_{s e}\left(X_{2}-X_{1}\right)+B_{2}\left(\dot{X}_{2}-\dot{X}_{1}\right)  \tag{1}\\
& K_{s e}\left(X_{2}-X_{1}\right)+B_{2}\left(\dot{X}_{2}-\dot{X}_{1}\right)=M\left(\ddot{X}_{1}-\ddot{X}_{6}\right)+K_{t}\left(X_{1}-X_{6}\right) \tag{2}
\end{align*}
$$

Substituting equation (1) into equation (2) to obtain the following equation
$F_{a g}=K_{l t} X_{2}+B_{1} \dot{X}_{2}+M\left(\ddot{X}_{1}-\ddot{X}_{6}\right)+K_{t}\left(X_{1}-X_{6}\right)$

Now
$K_{t}\left(X_{7}-X_{4}\right)+M\left(\ddot{X}_{7}-\ddot{X}_{4}\right)=K_{s e}\left(X_{4}-X_{3}\right)+B_{2}\left(\dot{X}_{4}-\dot{X}_{3}\right)$
$K_{s e}\left(X_{4}-X_{3}\right)+B_{2}\left(\dot{X}_{4}-\dot{X}_{3}\right)=F_{a n t}+K_{l t} X_{3}+B_{1} \dot{X}_{3}$

Substituting equation (4) into equation (5) to obtain the following equation
$K_{t}\left(X_{7}-X_{4}\right)+M\left(\ddot{X}_{7}-\ddot{X}_{4}\right)=F_{a n t}+K_{l t} X_{3}+B_{1} \dot{X}_{3}$

From equation (6)
$r M\left(\ddot{X}_{6}+\ddot{X}_{7}-\ddot{X}_{1}-\ddot{X}_{2}\right)+r K_{t}\left(X_{6}+X_{7}-X_{1}-X_{2}\right)=$
$J_{P} \ddot{\theta}+B_{P I}\left(\dot{\theta}-\dot{\theta}_{5}\right)+K_{P I}\left(\theta-\theta_{5}\right)$

Where

$$
\theta=57.296 \frac{X}{r}
$$

$\theta_{5}=57.296 \quad \frac{X_{5}}{r}$
Substituting to get:

$$
\begin{align*}
& r M\left(\ddot{X}_{6}+\ddot{X}_{7}-\ddot{X}_{1}-\ddot{X}_{2}\right)+r K_{t}\left(X_{6}+X_{7}-X_{1}-X_{2}\right)= \\
& J_{P}(57.296) \frac{\ddot{X}}{r}+B_{P 1}(57.296)\left(\frac{\dot{X}-\dot{X}_{5}}{r}\right)+K_{P 1}(57.296)\left(\frac{X-X_{5}}{r}\right) \tag{8}
\end{align*}
$$

But $J=J_{P} \frac{57.296}{r^{2}}, B_{3}=B_{P 1} \frac{57.296}{r^{2}}$ and $K_{1}=K_{P 1} \frac{57.296}{r^{2}}$
Therefore equation (8) becomes:

$$
\begin{align*}
& M\left(\ddot{X}_{6}+\ddot{X}_{7}-\ddot{X}_{1}-\ddot{X}_{2}\right)+K_{t}\left(X_{6}+X_{7}-X_{1}-X_{2}\right)=J \ddot{X}+B_{3}\left(\dot{X}-\dot{X}_{5}\right)+K_{l}\left(X-X_{5}\right)  \tag{9}\\
& \\
& \left.\left.B_{p 1}\left(\dot{\theta}-\dot{\theta}_{5}\right)+K_{p 1}\left(\theta-\theta_{5}\right)=B_{p 2} \dot{\theta}_{5}+K_{p 2} \theta_{5} \quad(10) \quad \ddot{X}_{1}-\ddot{X}_{6}\right)+K_{t}\left(X_{1}-X_{6}\right)-M\left(\ddot{X}_{7}-\ddot{X}_{4}\right)-K_{t}\left(X_{7}-X_{4}\right)\right)= \\
& L\left(J \ddot{X}+B_{3}\left(\dot{X}-\dot{X}_{5}\right)+K_{1}\left(X-X_{5}\right)\right)
\end{align*}
$$

$$
\begin{align*}
& \left.L / M\left(\ddot{X}_{1}+\ddot{X}_{4}-\ddot{X}_{6}-\ddot{X}_{7}\right)+K_{t}\left(X_{1}+X_{4}-X_{6}-X_{7}\right)\right\}= \\
& \left.L / J \hat{\tilde{X}}+B_{3}\left(\dot{X}-\dot{X}_{5}\right)+K_{1}\left(X-X_{5}\right)\right\} \tag{15}
\end{align*}
$$

$$
L\left\{M\left(\hat{\tilde{X}}_{1}+\hat{\tilde{X}}_{4}-2 \hat{\tilde{X}}\right)+K_{t}\left(\hat{X}_{1}+\hat{X}_{4}-2 \hat{X}\right)\right\}=
$$

$$
\begin{equation*}
L\left\{J \hat{\dot{X}}+B_{3}\left(\hat{\dot{X}}-\hat{\dot{X}}_{5}\right)+K_{1}\left(\hat{X}-\hat{X}_{5}\right)\right\} \tag{16}
\end{equation*}
$$

Using Laplace transformation to convert the
equations from time domain to $s$ domain yield

$$
\begin{equation*}
L\left\{\hat{F}_{a g}\right\}=L\left\{K_{l t} \hat{X}_{2}+B_{1} \hat{\dot{X}}_{2}+M(\hat{\tilde{X}})+K_{t}(\hat{X})\right\} \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
L\left\{B_{3}\left(\hat{\dot{X}}-\hat{\dot{X}}_{5}\right)+K_{1}\left(X-X_{5}\right)\right\}=L_{i}\left\langle B_{4} \hat{\dot{X}}_{5}+K_{2} \hat{X}_{5}\right\} \tag{18}
\end{equation*}
$$

$$
\begin{equation*}
L\left\{\hat{F}_{a n t}\right\}=L_{\{ }\left\{K_{t}(\hat{X})+M(\hat{\tilde{X}})-K_{l t} \hat{X}_{3}-B_{1} \hat{\dot{X}}_{3}\right\} \tag{13}
\end{equation*}
$$

where

$$
\begin{aligned}
& C_{6}=\left(2 B_{1} B_{3}+2 B_{1} B_{4}+B_{2} B_{3}+B_{2} B_{4}\right) M^{2} ; \\
& C_{5}=B_{1} B_{2} B_{34}(J+2 M)+\left(2 B_{1} K_{l}+2 B_{1} K_{2}+B_{2} K_{l}+B_{2} K_{2}+2 K_{s e} B_{3}+2 K_{s e} B_{4}\right. \\
& \left.+2 K_{l t} B_{3}+2 K_{l t} B_{4}+B_{2} B_{3}+B_{2} B_{4}\right) M^{2} \\
& C_{4}=M^{2}\left(2 K_{s e} K_{l}+2 K_{s e} K_{2}+2 K_{l t} K_{l}+2 K_{l t} K_{2}+B_{2} K_{l}+B_{2} K_{2}\right) \\
& +M\left(2 B_{1} B_{3} K_{l}+2 B_{1} B_{4} K_{l}+2 B_{2} B_{3} K_{l}+2 B_{2} B_{4} K_{l}+2 B_{2} B_{4} K_{t}+B_{l} B_{3} K_{t}+B_{l} B_{4} K_{t}\right. \\
& \left.+2 B_{1} K_{l}^{2}+2 B_{l} K_{l} K_{2}+2 B_{l} B_{2} K_{l}+2 B_{1} B_{2} K_{2}+2 B_{l} B_{3} K_{s e}+2 B_{l} B_{4} K_{s e}+2 B_{2} B_{3} K_{l t}\right) \\
& \quad B_{l} B_{2} B_{3}{ }^{2}+B_{l} B_{2} B_{3} B_{34}+J B_{l} B_{2} K_{l 2}+J B_{1} K_{s e} B_{34}+J B_{2} B_{3} K_{l t} \\
& C_{3}=M^{2}\left(2 B_{l} K_{l}^{2}+2 B_{l} K_{1} K_{2}+2 B_{2} K_{l}^{2}+2 B_{2} K_{l} K_{2}+2 B_{2} K_{l} K_{t}+2 B_{2} K_{2} K_{t}\right. \\
& +B_{l} K_{t} K_{l}+B_{l} K_{2} K_{t}+2 B_{3} K_{s e} K_{l}+2 B_{4} K_{s e} K_{l}+2 B_{3} K_{l t} K_{l}+2 B_{4} K_{l} K_{l t}+2 B_{3} K_{t} K_{s e} \\
& +2 B_{4} K_{t} K_{s e}+2 B_{3} K_{t} K_{l t}+2 B_{4} K_{t} K_{l t}+2 B_{l} K_{s e} K_{l}+2 B_{2} B_{4} K_{l t}+2 B_{2} K_{l} K_{l t}+2 B_{2} K_{l t} K_{2} \\
& \left.+2 B_{3} K_{s e} K_{l t}+2 B_{4} K_{s e} K_{l t}\right)+B_{3}\left(B_{l} B_{2} K_{l}+B_{l} B_{2} K_{2}+B_{l} B_{3} K_{s e}+B_{l} B_{4} K_{s e}+B_{2} B_{3} K_{l t}\right) \\
& +J\left(B_{1} K_{s e} K_{l}+B_{l} K_{s e} K_{2}+B_{2} B_{4} K_{l t}+B_{2} K_{l} K_{l t}+B_{2} K_{l t} K_{2}+B_{3} K_{s e} K_{l t}+B_{4} K_{s e} K_{l t}\right) \\
& +2 K_{t} B_{l} B_{2} B_{34}+K_{1} B_{l} B_{2} B_{34} 2 B_{l} B_{2} B_{3} K_{l}
\end{aligned}
$$

$$
\begin{aligned}
& C_{2}=M\left(2 K_{s e} K_{t} K_{l}+2 K_{s e} K_{t} K_{2}+K_{l t} K_{t} K_{l}+2 K_{l t} K_{t} K_{2}+2 K_{s e} K_{l}^{2}+2 K_{s e} K_{l} K_{2}+2 K_{l t} K_{l}^{2}\right. \\
& \left.+2 K_{l t} K_{l} K_{2}\right) \quad 2 K_{s e} K_{l t} B_{3}^{2} 2 B_{3} K_{l}\left(B_{l} K_{s e}+B_{2} K_{l t}\right)+[J+2 M]\left[K_{s e} K_{l t} K_{l 2}\right] \\
& +2 K_{l} K_{2}\left(B_{l} B_{3}+B_{l} B_{4}+B_{2} B_{3}+B_{2} B_{4}\right)+2 K_{t}\left(B_{l} B_{2} K_{l}+B_{l} B_{2} K_{2}+B_{l} B_{3} K_{s e}\right. \\
& \left.+B_{l} B_{4} K_{s e}+B_{2} B_{3} K_{l t}\right)+K_{l}\left(B_{l} B_{2} K_{l}+B_{l} B_{2} K_{2}+B_{l} B_{3} K_{s e}+B_{l} B_{4} K_{s e}+B_{2} B_{3} K_{l t}\right) K_{l}^{2} B_{l} B_{2} \\
& C_{l}=B_{3} K_{s e} K_{l t} K_{l 2}+2 K_{l} K_{t}\left(K_{s e} B_{3}+K_{s e} B_{4}+K_{l t} B_{3}+K_{l t} B_{4}\right) \\
& \quad K_{l}{ }^{2}\left(B_{l} K_{s e}+B_{2} K_{l t}\right)+2 K_{t}\left(B_{l} K_{s e} K_{l}+B_{l} K_{s e} K_{2}+B_{2} B_{4} K_{l t}+B_{2} K_{l} K_{l t}\right. \\
& \left.+B_{2} K_{l t} K_{2}+B_{3} K_{s e} K_{l t}+B_{4} K_{s e} K_{l t}\right)+K_{l}\left(B_{l} K_{l} K_{s e}+B_{l} K_{s e} K_{2}+B_{2} B_{4} K_{l t}+B_{2} K_{l} K_{l t}\right. \\
& \left.+B_{2} K_{2} K_{l t}+B_{3} K_{s e} K_{l t}+B_{4} K_{s e} K_{l t}\right) 2 B_{3} K_{l} K_{s e} K_{l t}+2 K_{l} K_{t}\left(K_{l} B_{l}+K_{2} B_{l}+K_{l} B_{2}+K_{2} B_{2}\right) \\
& C_{0}=2 K_{l} K_{t}\left(K_{l} K_{s e}+K_{2} K_{s e}+K_{l} K_{l t}+K_{2} K_{l t}\right) K_{l}^{2} K_{s e} K_{l t}+2 K_{t} K_{s e} K_{l t} K_{l 2}+K_{l} K_{s e} K_{l t} K_{l 2}
\end{aligned}
$$

Transforming back into the time domain yields
$M\left(2 B_{1}+B_{2}\right)\left(\dddot{F}_{a n t}\right)+M B_{2}\left(\dddot{F}_{a g}\right)+M\left(2 K_{s e}+K_{l t}\right)\left(\ddot{F}_{a n t}\right)+M K_{s e}\left(\ddot{F}_{a g}\right)$
$+K_{t}\left(2 B_{l}+B_{2}\right)\left(\dot{F}_{a n t}\right)+B_{2} K_{t}\left(\dot{F}_{a g}\right)+K_{t}\left(2 K_{s e}+K_{l t}\right)\left(\dot{F}_{a n t}+K_{t} K_{s e}\left(\dot{F}_{a g}\right)\right.$
$=C_{6} \dddot{X}+C_{5} \dddot{X}+C_{4} \dddot{X}+C_{3} \dddot{X}+C_{2} \ddot{X}+C_{1} \dot{X}+C_{0}$

Converting from $X$ to $\theta$ gives
$\left[\begin{array}{l}\delta\left(\left(2 B_{1}+B_{2} B\right) M \dddot{F}_{a n t}+M B_{2} \dddot{F}_{a g}+\left(2 K_{s e}+K_{l t}\right) M \ddot{F}_{a n t}+M K_{s e} \ddot{F}_{a g}+K_{t}\left(2 B_{1}+B_{2}\right) \dot{F}_{a n t}\right. \\ \left.+B_{2} K_{t} \dot{F}_{a g}+K_{t}\left(2 K_{s e}+K_{l t}\right) F_{a n t}+K_{t} K_{s e} F_{a g}\right)=\dddot{\theta}+P_{5} \dddot{\theta}+P_{4} \dddot{\theta}+P_{3} \dddot{\theta}+P_{2} \ddot{\theta}+P_{1} \dot{\theta}+P_{0} \theta\end{array}\right]$
$\delta=\frac{57.296}{r C_{6}}, P_{5}=\frac{C_{5}}{C_{6}}, P_{4}=\frac{C_{4}}{C_{6}}, P_{3}=\frac{C_{3}}{C_{6}}, P_{2}=\frac{C_{2}}{C_{6}}, P_{1}=\frac{C_{1}}{C_{6}}, P_{0}=\frac{C_{0}}{C_{6}}$
Based on an analysis of experimental data, parameter estimates for the oculomotor plant are [5];
$K_{s e}=125 \mathrm{Nm}^{-1} ; K_{l t}=60.7 \mathrm{Nm}^{-1} ; B_{1}=2.0 \mathrm{Nsm}^{-1} ; B_{2}=0.5 \mathrm{Nsm}^{-1} ; J=2.2 \times 10^{-3} \mathrm{Ns}^{2} \mathrm{~m}^{-1} ;$
$B_{3}=0.538 \mathrm{Nsm}^{-1} ; B_{4}=41.54 \mathrm{Nsm}^{-1} ; K_{1}=26.9 \mathrm{Nm}^{-1} ; K_{2}=41.54 \mathrm{Nm}^{-1}$ and $r=0.0118 \mathrm{~m}$.
In this modifieid model, the mass of eye is $[M=28.34952 \mathrm{~g}]$, and the elasticity of tendon assumed to be equal to $K_{t}=1 \mathrm{Nm}^{-1}$ to show their effects on the model of Bronzeno [5]
So;
$C_{6}=152182.691 ; C_{5}=1287215.62 ; C_{4}=21307704.7016 ; C_{3}=31186435.2448$
$C_{2}=50012985.3697 ; C_{1}=10304263.896 ; C_{0}=21181589.3$
and
$P_{5}=8.4584 ; P_{4}=140.014 ; P_{3}=204.927 ; P_{2}=328.6378 ; P_{1}=67.71 ; P_{0}=139.185 ; \delta=0.03190635$
$\because \theta=\frac{57.296 X}{r} \quad \therefore X=\frac{\theta . r}{57.296}$
Another value of elasticity of tendon was assumed to be $K_{t}=50 \mathrm{Nm}^{-1}$.

$$
\begin{aligned}
& C_{6}=152182.6591 ; C_{5}=1287215.62 ; C_{4}=21604995.06 ; C_{3}=53420588.19 ; C_{2}=78328428.76 ; \\
& C_{1}=64824525.58 ; C_{0}=94595358.3
\end{aligned}
$$

> And
> $P_{5}=8.4584 ; P_{4}=141.9675224 ; P_{3}=315.0294044 ; P_{2}=514.700092 ;$
> $P_{1}=425.9652576 ; P_{0}=621.5909149 ; \delta=0.03190635$

## Controller of the Modified Eye Model

According to the modified eye model signals, the PI controller of this model was designed to determine whether this system enhances the signal or not. The result explain the signal of the PI controller in the normal case and during the eye disease and the effect of each on the system. When the elasticity of tendon is assumed to be equal to $K_{t}=1 \mathrm{Nm}^{-1}$ and
$K_{t}=50 \mathrm{Nm}^{-1}$ during normal case and during eye disease case they are applied and processed with SIMULINK of MATLAB to get the desired theta, velocity and acceleration through closed loop control system. In the block diagram below $\theta, \dot{\theta}, \ddot{\theta}$ refer to position, velocity and acceleration by integration used in MATLAB as shown in Fig.(4).


Fig.(4) SIMULINK block diagram of controller.

## Results and Discssion

The following results was obtained when the agonist and antagonist tension was applied to the system eye model. Fig. (5) and (6), quantify the relationship for agonist input and the antagonist input.

With, $F_{p}=1.3 \mathrm{~N}, t_{1}=0.1 \mathrm{sec}, t_{a c}=0.018 \mathrm{sec}$ and $t_{d e}=0.018 \mathrm{sec}_{P_{1}}=67.71 ; \quad P_{o}=139.185 ; \delta=0.03190635$

And the elasticity of tendon is assumed to be $K_{t}=1 \mathrm{Nm}^{1}$

So;

$$
\begin{aligned}
& C_{6}=152182.659 ; C_{5}=1287215.62 C_{4}=21307704.016 ; \\
& C_{3}=31186435.248 ; C_{2}=50012985.897 ; C_{1}=10304263.86 \\
& C_{0}=21181589.3
\end{aligned}
$$

And

$$
P_{5}=8.4584 ; P_{4}=140.014 ; P_{3}=204.927 ; P_{2}=328.6378
$$

$$
=67.71 ; \quad P_{O}=139.185 ; \delta=0.03190635
$$

Using equation (20) to obtaine Fig.(5)-Fig.(9). When applying the agonist and antagonist active state tension, the position response of the eye model (theta) is shown in Fig.(7). And the velocity response of the eye model is shown in Fig.(8). Finally the acceleration response of the eye model is shown in Fig.(9).


Fig.(5) Agonist active-state tension


Fig.(7) Theta response of the eye model


Fig.(6) Antagonist active-state tension


Fig.(8) Angualr velocity response of the eye model


Fig.(9) Acceleration response of the eye model.

Theta response $(\theta)$ in the modified eye model is very small due to the elasticity of tendon $K_{t}$ which is applied to it. When the modified eye model responds to the applied force after 0.4 second, the velocity achieves the higher value and this is normal because the velocity and acceleration depend up on theta ( $\theta$ ) and is affected by any difference occurring n theta $(\theta)$.

By comparing the modified eye model with the truer linear homeomorphic saccadic eye movement model, the signals of theta, velocity and acceleration are different because of the
adding content of the modified eye movement model which is the elasticity of tendon and the mass of eye that gives a theta ( $\theta$ ) with sixth order, so the system needs a greater force than that used in the truer linear homeomorphic saccadic eye movement model to move with $15^{\circ}$. This is impossible because this force comes as a result of brain signal to the eye and giving it order to move to the desired angle for this a PI controller was designed to enhance the system with execution time ( t ) $=1 \mathrm{sec}$ as shown in Fig.(10), (11) and (12). The Acceleration response of the eye model with PI controller is shown in Fig.(12).


Fig.(10) Theta response of the eye model with PI controller


Fig.(11) Velocity response of the eye model with PI controller


Fig.(12) Acceleration response of the eye model with PI controller

When the disease affects the eye muscle, the elasticity $K_{s e}$ and the viscosity $B_{2}$ of eye muscle are affected too.
The following signals of eye model show the effect of lateral rectuse muscle disease on the behavior of system and how it differs from the eye movement model in the primary position without disease.
There are two types of effect, the increasing and decreasing of the elasticity $K_{s e}$ and the viscosity $B_{2}$ of eye muscle was taken in the truer linear homeomorphic saccadic eye movement model and modified eye movement model.


Fig.(13) Theta response of truer linear homeomorphic
saccadic eye movement model with decreasing of
Fig.(13) Theta response of truer linear homeomorphic
saccadic eye movement model with decreasing of elasticity $K_{s e}$ and the viscosity $B_{2}$

Fig.(13) shows the effecet of decreasing the elasticity $K_{\text {se }}$ (which is estimated to be 100 $\mathrm{Nm}^{-1}$ ) and the viscosity $B_{2}$ (which is estimated to be $0.25 \mathrm{Nsm}^{-1}$ ) of homeomorphic saccadic eye movement model, on the response of theta.
Fig.(14) shows the effect of decreasing the elasticity $K_{s e}$ (which is estimated to be 100 $\mathrm{Nm}^{-1}$ ) and the viscosity $B_{2}$ (which is estimated to be $0.25 \mathrm{Nsm}^{-1}$ ) of homeomorphic saccadic eye movement model on the response of velocity.


Fig.(14) Velocity response of truer linear homeomorphic saccadic eye movement model with decreasing of elasticity $K_{s e}$ and the viscosity $B_{2}$

Fig.(15) shows the effect of decreasing in the elasticity $K_{s e}$ (which is estimated to be 100 $\mathrm{Nm}^{-1}$ ) and the viscosity $B_{2}$ (which is
estimated to be $0.25 \mathrm{Nsm}^{-1}$ ) of homeomorphic saccadic eye movement model on the response of acceleration.


Fig.(15) Effect of decreasing of elasticity $K_{s e}$ and the viscosity $B_{2}$ on the acceleration response of truer linear homeomorphic saccadic eye movement model with

The signal of theta, velocity and acceleration when the disease affects the eye muscle is decreased because of the compression of the eye muscle which decreases the overshoot of truer linear homeomorphic saccadic eye movement because the elasticity $K_{s e}$ decreases which means the effect of any force applied to it does not cause the same movement when it is in the normal case without effect of disease. The increase in the elasticity $K_{s e}$ which is estimated to be $150 \mathrm{Nm}^{-1}$ and the viscosity
$B_{2}$ which is estimated to be $0.75 \mathrm{Nsm}^{-1}$ of homeomorphic saccadic eye movement model, Fig.(16) shows the effect of this increase on the response of theta.
Fig.(17) shows the effect of increasing the elasticity $K_{\text {se }}$ (which is estimated to be 150 $\mathrm{Nm}^{-1}$ ) and the viscosity $B_{2}$ (which is estimated to be $0.75 \mathrm{Nsm}^{-1}$ ) of homeomorphic saccadic eye movement model on the response of velocity.


Fig.(16) Theta response of truer linear homeomorphic saccadic eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$

Fig.(18) shows the effect of increasing the elasticity $K_{\text {se }}$ (which is estimated to be 150
$\mathrm{Nm}^{-1}$ ) and the viscosity $B_{2}$ (which is Nm ${ }^{-1}$ )


Fig.(17) Velocity response of truer linear homeomorphic saccadic eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$ estimated to be $0.7 \mathrm{Nsm}^{-1}$ ) of homeomorphic saccadic eye movement model on the response of acceleration.


Fig.(18) Acceleration response of truer linear homeomorphic saccadic eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$

In the case of increasing, the difference between figures of the model during the effective disease and without disease is clear because the eye muscle is extended which means the force affecting to it $F_{a n t}$ and $F_{a g}$ make the eye move continuously without any control to it. Also the time delay is very high as shown in the previous figures which show the signal has achieved the settling time after ten seconds while in the normal case it achieves the settling time approximately after two seconds,so this disease because of the extension of the eye muscle tends to cause a high time delay and this is a bad thing to the


Fig.(19) Theta response of modified eye movement model with decreasing of elasticity $K_{s e}$ and the viscosity $B_{2}$

Fig.(20) shows the decrease in the elasticity $K_{s e}$ which is estimated to be $100 \mathrm{Nm}^{-1}$ and the viscosity $B_{2}$ which is estimated to be 0.25
system because we need reaching the desired angle with minimum possible time regarding the settling time of any system which must be as minimum as possible and this disease tends to increase the settling time, also the
oscillation of the acceleration is clear and it is unstable because of the extension and failure of eye muscle.
In the modified eye movement model the elasticity $K_{s e}$ decreases which is estimated to be $100 \mathrm{Nm}^{-1}$ and the viscosity $B_{2}$ which is estimated to be $0.25 \mathrm{Nsm}^{-1}$, Fig.(19) shows the response of this decrease on theta.
$N s m^{-1}$ of modified eye movement model which affect the response of velocity.
Fig.(21) shows the decrease in the elasticity $K_{s e}$ which is estimated to be $100 \mathrm{Nm}^{-1}$ and the viscosity $B_{2}$ which is estimated to be $0.25 \mathrm{Nsm}^{-1}$ of modified eye movement model which affect the response of acceleration.


Fig.(20) Velocity response of modified eye movement model with decreasing elasticity $K_{s e}$ and the viscosity $B_{2}$


Fig.(21) Acceleration response of modified eye movement model with decreasing of elasticity $K_{s e}$ and the viscosity $B_{2}$
Another value of the elasticity $K_{s e}$ decreases was taken which is estimated to be $85 \mathrm{Nm}^{-1}$ and the viscosity $B_{2}$ which is estimated to be $0.1 \mathrm{Nsm}^{-1}$, figures (22), (23), and (24) show the response of this decrease on theta, velocity and acceleration.

The decreasing of elasticity $K_{s e}$ and the viscosity $B_{2}$ of the modified eye movement model makes the eye muscle contract and this contraction the muscle reaches a saturation value so any increase in the force does not affect the output and the increasing of he force makes the spring tend to fail.

In the case of increase in the elasticity $K_{s e}$ is estimated to be $150 \mathrm{Nm}^{-1}$ and the viscosity $B_{2}$ is estimated to be $0.75 \mathrm{Nsm}^{-1}$ of the modified eye movement, Fig.(25) shows the effect of this increase on theta response


Fig.(22) Theta response of modified eye movement model with decreasing of elasticity $K_{s e}$ and the viscosity $B_{2}$


Fig.(24) Acceleration response of modified eye movement model with decreasing of elasticity $K_{s e}$ and the viscosity $B_{2}$


Fig.(25) Theta response of modified eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$

Fig.(26) shows the increase in the elasticity $K_{s e}$ which is estimated to be $150 \mathrm{Nm}^{-1}$ and the viscosity $B_{2}$ which is estimated to be 0.75 $N s m^{-1}$ of modified eye movement model which affect the response of velocity.
Fig.(27) shows the increase in the elasticity $K_{s e}$ is estimated to be $150 \mathrm{Nm}^{-1}$ and the viscosity $B_{2}$ is estimated to be $0.75 \mathrm{Nsm}^{-1}$ of modified eye movement model which affect the response of acceleration.


Fig.(26) Velocity response of modified eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$


Fig.(27) Acceleration response of modified eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$

When another value of the increasing in the elasticity $K_{s e}$ was taken which is estimated to be $165 \mathrm{Nm}^{-1}$ and the viscosity $B_{2}$ is estimated to be $0.65 \mathrm{Nsm}^{-1}$ of the modified eye movement, Fig.(28) shows the effect of this increase on theta response.

During the increase in the elasticity $K_{\text {se }}$ and the viscosity $B_{2}$ of modified eye movement model, the muscle was extended and as shown in the previous figures, the system of the modified eye movement model starts to respond after six seconds and for this response theta oscillates and is unstable for a desired value and if the larger time is taken in MATLAB, the same result appears which is the theta value unstable for a desired range with continuous increasing and decreasing.

In Fig.(30) the system begins to respond to the applied force after 6.5 second and the value of velocity oscillates with increasing and decreasing of its value and cannot be stable for a desired value. This disease makes the increase in the elasticity $K_{s e}$ and the viscosity $B_{2}$ for both eye model tend to destabilize of theta, velocity and acceleration in addition to the high time delay that happens


Fig.(28) Theta response of modified eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$


Fig.(29) Velocity response of modified eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$


Fig.(30) Acceleration response of modified eye movement model with increasing of elasticity $K_{s e}$ and the viscosity $B_{2}$

## Conclusions

After the mathematical model is made by using SIMULINK of MATLAB program, the response of eye model is found.

- Through the mechanical model we not get the desired angle however the PI controller is to get the desired angle but the velocity and acceleration begin to slow down while they should be equal to zero when the desired angle is achieved.
- Theta response $(\theta)$ in the modified eye model is very small due to the elasticity of tendon $K_{t}$ which is applied to it. When the modified eye model responds to the applied force after 0.4 seconds, the velocity achieves the higher value and this is normal because the velocity and acceleration depend up on theta ( $\theta$ ) and are affected by any difference in the value of theta ( $\theta$ ).
- The signal of theta, velocity and acceleration when the disease affects the eye muscle decreases because of the compression of the eye muscle which decreases the overshoot of truer linear homeomorphic saccadic eye movement
- The signal of theta, velocity and acceleration when the disease affects the eye muscle
decreases because of the compression of the eye muscle.
- By comparing the eye movement during disease and without it, it is clear because the eye muscle is extended or compressed causing increase or decrease in the primary elastic element $K_{s e}$ and the viscosity $B_{2}$ respectively which means in the case of increasing the primary elastic element $K_{\text {se }}$ and the viscosity $B_{2}$,the force affecting
- ( $\mathrm{F}_{\text {ant }}$ and $\mathrm{F}_{\mathrm{ag}}$ ) makes the eye move continuously without any control over it.
- The time delay is very high.

Also the oscillation of the acceleration is clear and it is unstable due to extension and failure of eye muscle but in the case of decreasing the primary elastic element $K_{s e}$ and the viscosity $B_{2}$, the muscle reaches a saturation value so any increase in the force not affect the output and the increase in the force makes the spring tend to fail.

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## الخلاصة

يشهد العلم اليوم تطور اً رئيساً ومتز ايد بسر عة عالية في الأختر اعات والأبداعات في كافة الحقول
 المشروع وتوضـح تصـيم الحركـة الميكانيكيـة للعين البشـرية و الذي يتضـمن ثـلاث حركـات هـي الأساسية والثانوية والثلاثية.

تم تطوير نموذج لحركة العين الميكانيكيـة الاحيائيـة باضـافة وحسـاب كتلـة العين ومعامل مرورنــة وتر العين في حالة حركـة العين الابتدائيـة. قد ظهرت طبقا لهذا النطوير، معـادلات مـن الارجـة السادسة والتي عولجت باستخدام MATLAB SIMULINK للحصـول على النتائج المطلوبـة للزاوية والسر عة والتعجيل في الحركة الطبيعية.
تم أخذ النتائج لنفس النموذج في دلـي تتاثثر بالمرض الذي تم افتر اضه حيث لوحظ كون زيادة ونقصـان تصيب معامل المرونة الابتدائيـة للعين ولزوجة العين. بعد ذلك تم اضافة مسيطر للموديل ليتم السيطرة على النظـام الحركي للعين
 المسيطر الذي تم استعماله اسنطاع فقط السيطرة على النظام الحركي للعين بالحالـة الطبيعيـة بينمـا في حالة المرض فالمسيطر لم يستطع السيطرة على النظام الحركي للعين للحصـول على الزاويـة المطلوبة.

