Design of Integral Sliding Mode Controller for Servo DC Motor

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Abstract

DC servo motor is simple in construction and control and has many applications. However, the uncertainties due to its parameters changes such as load torque and friction are an evitable. Therefore, a robust controller has to be employed for keeping specified requirements irrespective to parameter variations. In the present work, two sliding mode controllers have been suggested to control the speed of DC motor under motor load changes; classical and integral sliding mode controllers. The integral slide mode control could show better tracking characteristics than its counterpart and also could compensate the change in system parameters.

Keywords: DC Motor, integral sliding control, speed control, sliding mode control.

1 Introduction

Nowadays DC servo systems have huge industrial applications ranges from controlling the tray of a CD or DVD player, electric vehicles, steel rolling mills, electric cranes, robotic manipulators, to different home appliances. The reason behind such many applications is due to precise, wide, simple, and continuous control characteristics [1, 2].

Moreover, DC motor has the feature of quadrature axes of both torque and flux. This would result in high torque to current ratio and thus increasing motor efficiency [3, 4].

It has been shown that sliding mode control is an efficient tool for controlling complex nonlinear and high-order dynamic plants operating under uncertainty conditions, which is a challenging problem for most modern control strategies. This reveals the high level of researches and publication activities in this control field and the remitting interest of practicing engineers in sliding mode control during the last two decades. Automotive systems are typical among such applications. They possess complicated nonlinearities and are subject to significant model uncertainties and external disturbances. A variety of automotive control applications have benefited greatly from sliding mode techniques in terms of control accuracy and robustness [5, 6, 7].

The key idea of designing VSS control algorithms is to enforce sliding mode in certain manifold of state space. These manifolds are formed by the intersection of hyper-surfaces in the state space. As such, an intersection domain will arise which is usually called switching manifold. As soon as, the system state trajectory touches the switching plane, the feedback loop structure is adaptively changed such that the system state is forced to slide along the switching plane. Thereafter, the system response relies on switching plane gradient and under what is a so-called matching condition; the solution would be insensitive to variations of system parameters and external disturbances [5, 6, 7].

The concept of integral sliding mode is an extension of classical sliding mode scheme. The integral sliding mode focuses on the robustness over the entire system response. In this new type of sliding mode, the order of the motion equation is identical to plant model dimension.. This indicates that the sliding mode is established without a reaching phase and, starting from the initial time instant, the invariance of the system is guaranteed against parametric uncertainty and external disturbances [8, 9].

2 Mathematical model of permanent magnet DC motor

The permanent magnet DC motor mathematical model can be described in the equation below

$$\ddot{\omega} = -\left(\frac{R_a}{L_a}+\frac{b}{J}\right)\dot{\omega} - \left(\frac{bR_a + k_m k_b}{JL_a}\right)\omega + \frac{k_m}{JL_a}u - \frac{R_a}{JL_a}d \qquad (1)$$

where ω is the armature angular velocity (rad/sec), R_a and L_a are the armature resistance (ohm) and inductance (Henry) respectively, b and J are damping friction coefficient (N·m·sec/rad) and moment of inertia (kg·m^2) respectively, k_m and k_b are the motor torque and back emf constants respectively. U is the armature voltage (volts) and d is the external load torque (N·m).

Let us define $e_1 = \omega - \omega_d$, $e_2 = \dot{\omega} - \dot{\omega}_d$ where ω_d represent the desired angular velocity (rad/sec). Then the system described by Eq.(1) can be represented in the state variables as below:

$$\dot{e}_1 = e_2$$
$$\dot{e}_2 = a_2 e_2 + a_1 e_1 + b_2 u + h_2 d - \ddot{\omega}_d \quad (2)$$

where

$$a_2 = -\left(\frac{R_a}{L_a} + \frac{b}{J}\right),\ a_1 = -\left(\frac{bR_a + k_m k_b}{JL_a}\right),$$
$$b_2 = \frac{k_m}{JL_a}, h_2 = -\frac{R_a}{JL_a}$$

3 Sliding mode controller design for speed control

Assuming that $d = 0$ and define a surface in the state space

$$s = e_2 + ce_1 \quad (3)$$

If a controller u was designed to make the system trajectories head to the surface $s = 0$, then Eq. (2) can be written as,

$$e_2 + ce_1 = 0$$
$$\dot{e}_1 + ce_1 = 0 \quad (4)$$

The time solution for the equation above is written as

$$e_1(t) = e_1(0)e^{-ct} \quad (5)$$

The equation above shows that if the state trajectories are forced to move on surface $s = 0$, then e_1 will tend to zero exponentially after a finite time interval or one can say

$$e_1(t = \infty) = 0 \to \omega = \omega_d \quad (6)$$

Thus, if the state trajectories are forced to move on surface $s = 0$ then the motor will reach the desired angular velocity exponentially within a specified time interval.

To ensure that the state trajectories will head toward the surface $s = 0$, the following reaching condition should be fulfilled [5,8]

$$s\dot{s} < 0 \quad (7)$$
$$\dot{s} = \dot{e}_2 + c\dot{e}_1$$
$$s\dot{s} = s\,[\dot{e}_2 + c\dot{e}_1]$$
$$s\dot{s} = s\,[a_2 e_2 + a_1 e_1 + b_2 u + ce_2] \quad (8)$$

If one assume that the control defined as a discontinuous function for the surface s as below

$$u = -k * sign(s) \quad (9)$$

Substitute for the control u in Eq. (9)

$$s\dot{s} = s\,[a_2 e_2 + a_1 e_1 - b_2\, k\, sign(s) + ce_2]$$
$$s\dot{s} = sa_2 e_2 + sa_1 e_1 - sb_2 k sign(s) + s\,c\,e_2$$

Using the fact that $s * sign(s) = |s|$ and from the inequality properties (see calculus) that $ab \le |a||b|$ results in the following;

$$s\dot{s} \le |s|[|a_2||e_2| + |a_1||e_1| - b_2 k + |c||e_2|] < 0 \quad (10)$$

Solving for k

$$k > \frac{|a_2||e_2| + |a_1||e_1| + |c||e_2|}{b_2} \quad (11)$$

4 Integral sliding mode control for disturbance rejection

Let the DC motor system with the load torque be described in the following form

$$\dot{\boldsymbol{e}} = \boldsymbol{f}(e,t) + \boldsymbol{g}(e,t)\, u + \boldsymbol{h}\,(e,t)\, d \quad (12)$$

where $e = [e_1\ e_2]$

$$\boldsymbol{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ a_2 e_2 + a_1 e_1 - \ddot{\omega}_d \end{bmatrix},$$
$$\boldsymbol{g} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{k_m}{JL_a} \end{bmatrix},$$
$$\boldsymbol{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{R_a}{JL_a} \end{bmatrix} d$$

Here $h(e,t)$ is the disturbance function caused by external load and disturbance torques. The main idea of integral sliding mode is to design the control law as two parts; a stabilizing component which described above and the second component is designed to reject the disturbance. Taking this assumption into account we can rewrite the control law as follows

$$u = u_0 + u_1 \quad (13)$$

where u_0 is designed to make the system follows a specified trajectory e_0 in the state space and u_1 is designed to cancel the disturbance. From Eq.(12), Eq.(13) becomes

$$\dot{e} = f(e,t) + g(e,t)u_0 + g(e,t)u_1 + h(e,t)d \quad (14)$$

Furthermore the surface s can be written in the following form [5,8]

$$s = s_0 + z \quad (15)$$

where z represents the integral part which cancels the disturbance.

In order to keep the system on the specified trajectory s_0 the time derivative of the surface s should equal zero. In other words, the system trajectory will not leave the surface s_0; i.e,

$$\dot{s} = \dot{s}_0 + \dot{z} = 0 \quad (16)$$

Since,

$$s_0 = e_2 + ce_1 = C^T e$$

Then,

$$\dot{s}_0 = \dot{e}_2 + c\dot{e}_1 = C^T \dot{e} \quad (17)$$

Using Eq.(14), the above equation becomes;

$$\dot{s}_0 = C^T[f(e,t) + g(e,t)\, u_0 + g(e,t)\, u_1 + h(e,t)\, d]$$

Accordingly, Eq.(16) can be written as,

$$\dot{s} = C^T\, [f(e,t) + g(e,t)\, u_0 + g(e,t)\, u_1 + h(e,t) d\,] + \dot{z} = 0 \qquad (18)$$

In order ensure that $e(t) = e_0(t)$ for all $t > 0$ the following condition should be fulfilled

$$g(e,t)\ \ u_1 = h(e,t)\, d$$

Substitute in $\dot{s}$ we get

$$\dot{z} = -C^T[f(e,t) + g(e,t)u_0] \qquad (19)$$

Integrating Eq,(19), one can get

$$z = -C^T \int_{t_0}^{t} [f(e,t) + g(e,t)\, u_0]\ dt \qquad (20)$$

Thus, substitution for z is the surface equation s will ensure that the system trajectories will remain on the surface s even under external disturbances. If u_1 considered to be a nonlinear function

$$u_1 = -k_1 sign(s) \qquad (21)$$

Then the final control law be rewritten as follows

$$u = u_0 + u_1$$

$$u = -k_0 sign(s_0) - k_1 sign(s) \qquad (22)$$

In what follows a sufficient condition will be established. It has been mentioned earlier that to have a disturbance rejection then the following condition should be fulfilled

$$g(e,t)\, u_1 = h(e,t)\, d \qquad (23)$$

Substitute for u_1 in Eq.(23), we have

$$-g(e,t)\, k_1\ sign\ (s) = h(e,t)\, d \qquad (24)$$

Taking absolute values for the two sides yields

$$k_1 \geq \left|\frac{h(e,t)}{g(e,t)} d_{max}\right| \qquad (25)$$

5 Simulated Results

The system and design parameters are listed in Table (1).

Table (1) System and Design Parameters

Parameter	value
Armature resistance R_a	5 Ω
Armature inductance L_a	0.5 H
Damping coefficient b	0.01 N.m.s/rad
Moment of inertia J	0.001 Kg.m^2
Torque constant K_m	0.625 N.m/amp
EMF constant K_b	0.001 volt.sec/rad
k_0	8.1
k_1	5

The modeling and simulation of classical and integral slide mode has been performed using MATLAB/Simulink. The simulation includes MATLAB function blocks which be written in m-code. This class of function would allow information from Simulink software to be passed into and also provide information to Simulink software. The Simulink model for sliding mode controller is shown in Fig.(1). The SIMULINK modeling of integral sliding mode control of DC motor consists of two main function blocks entitled "DC motor model" and "Integral sliding mode control". The filter is used to smooth out the control signal for measuring the average input signal. The function block named current estimator is used to give armature current.

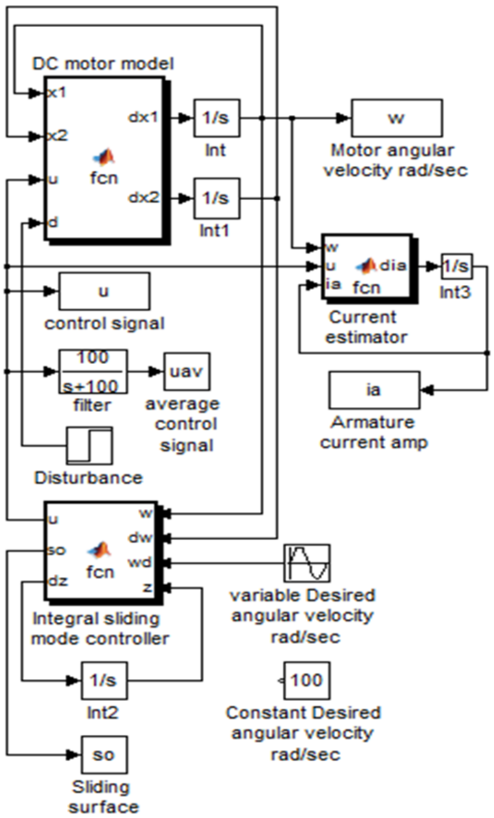

Figure (1) Simulink Modeling of Sliding Mode Controller

Figure (2) shows the behaviors of different system variables due to sinusoidal input without disturbance. A sinusoidal input of 100 rad/sec. is fed as an input to the system. The non-integral slide mode controller is used in this scenario. It is clear from the results that tracking of input is achieved after 4 seconds. Although the controller gives good tracking performance, perfect coincidence could not be achieved; as there is still small error of different values along the speed trajectory. The actuating signal of sliding-mode controller shows high frequency on-off characteristics as shown in Fig.(2.c). The filtered version of the actuating signal is depicted in Fig.(2.d). The sliding surface and phase plane of error e_1 and change of error e_2 is illustrated in Fig.(2.e) and Fig.(2.f), respectively.

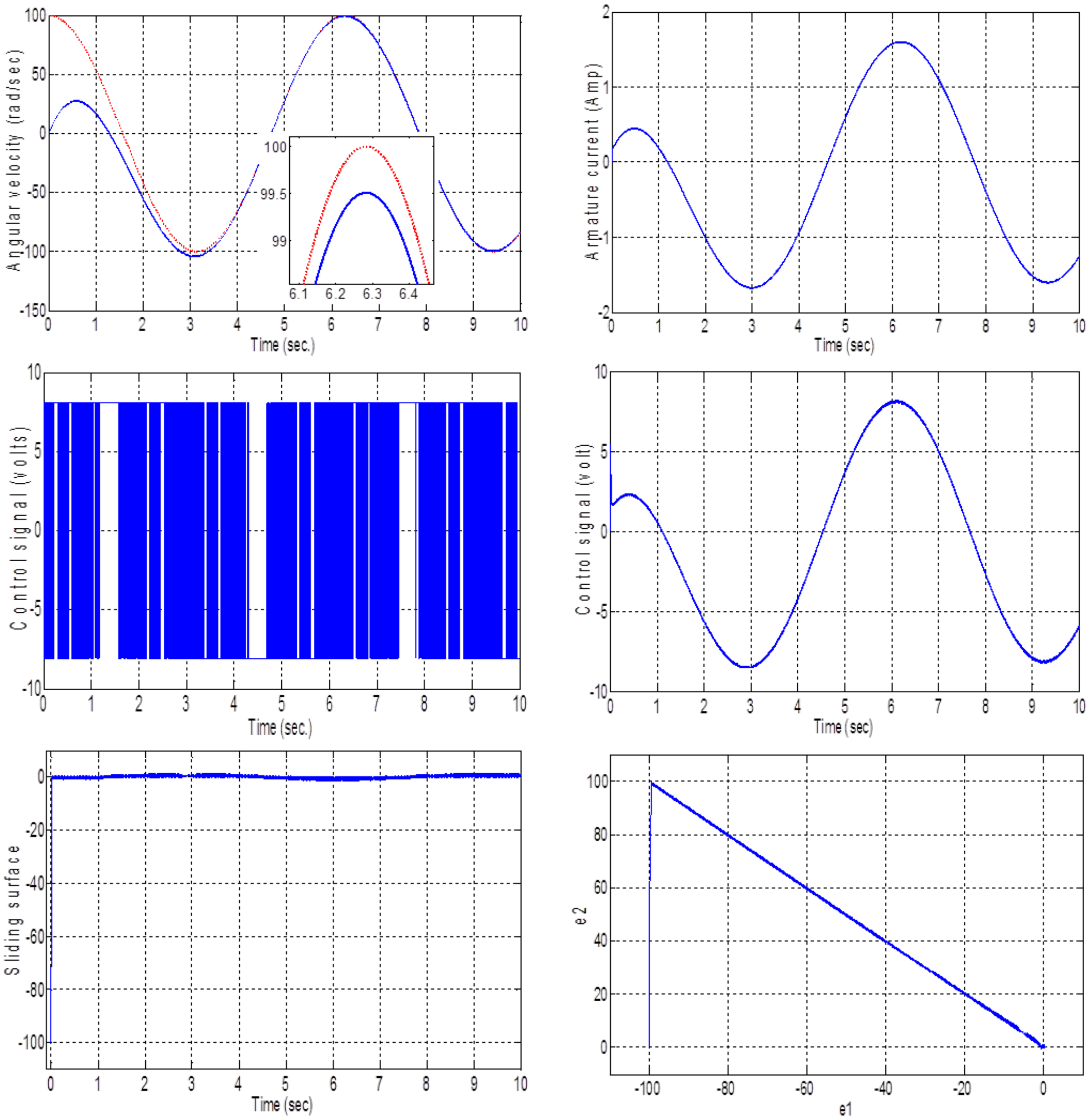

Figure (2) Simulation of different system responses due to sinusoidal input without disturbance
(a) Velocity response (b) Current behavior (c) Non-filtered actuating signal
(d) Filtered actuating signal sliding surface behavior (f) phase plane of $e_2 - e_1$

The above scenario has been repeated with a disturbance of constant amplitude (0.5 N.m) is exerted. The system responses due to this disturbance are shown in Fig.(3). It is evident from the figure there is great degradation in speed response and the classical sliding mode controller could not compensate such parameter change of the system

Figure (4) shows the different behaviors when integral sliding mode strategy is included in control instead of classical slide mode control. It is clear that for the same disturbance application, the speed response shows good characteristics and tracking to such disturbance. Therefore, one can conclude that the integral slide mode control is more robust than classical one.

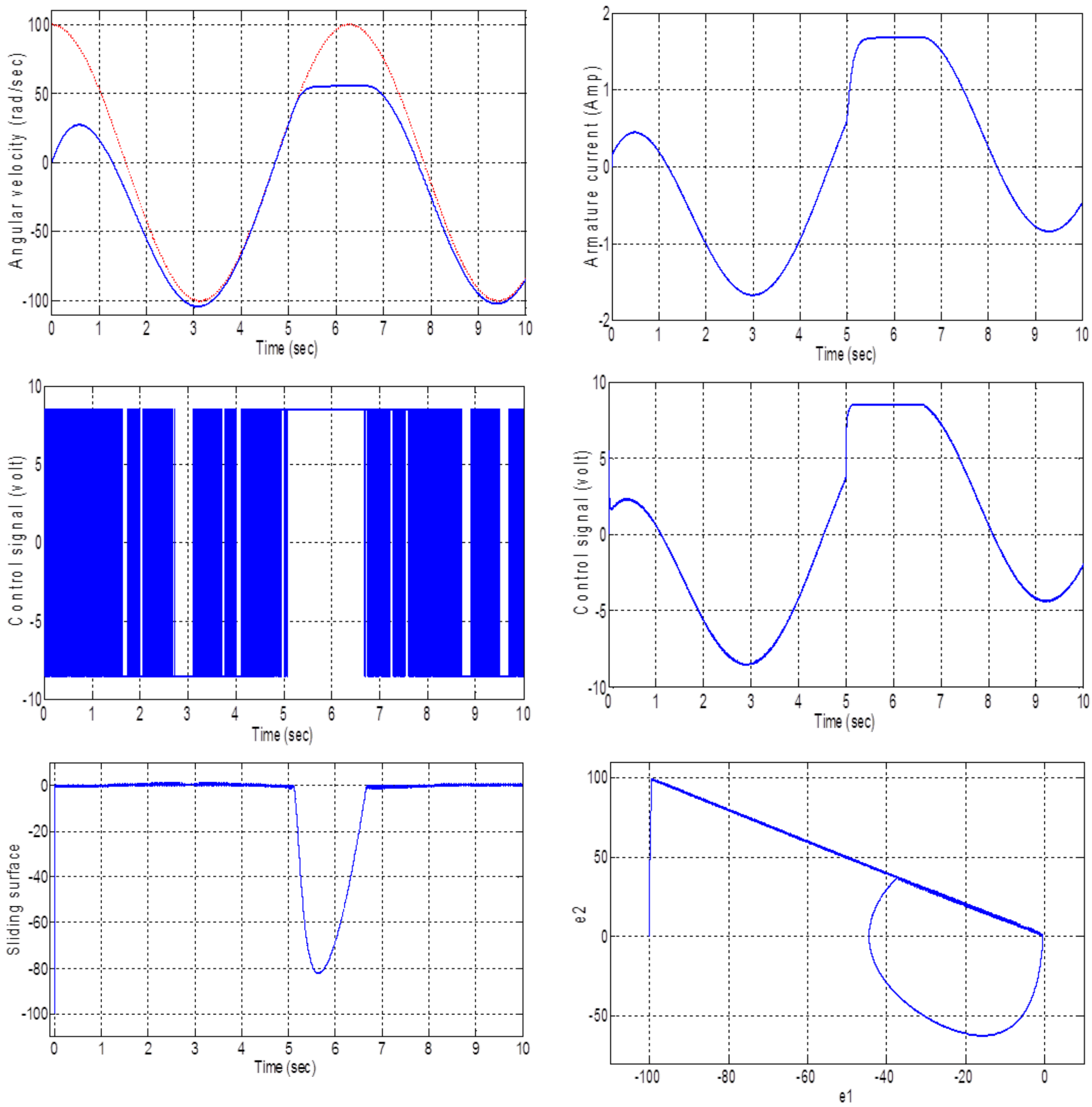

Figure (3) Simulation of different system responses due to sinusoidal input without disturbance
(a) Velocity response (b) Current behavior (c) Non-filtered actuating signal
(d) Filtered actuating signal & sliding surface behavior (f) phase plane of $e_2 - e_1$

6 Conclusion

In this work, the performance of two sliding mode control strategies has been assessed in terms of their capabilities to reject the applied disturbance.

In spite that the classical sliding mode controller shows satisfactory tracking performance, as indicated in Fig.(2), the response resulting from this controller would degrade due to disturbance exertion. As such, the classic controller could not compensate the change in system parameter as shown in Fig.(3).

On the other hand, the integral sliding mode controller has better disturbance rejection capability than its opponent when the same load change is applied to DC controlled system. However, the price of better disturbance rejection capability is the increase in input voltage which has to overcome this abrupt load change, as illustrated in Fig.(4).

Thereby, the simulated results have shown that the integral sliding control outperforms the classical sliding control and gives better transient and tracking performance. The integral sliding mode control has the ability to reject the disturbance and then it is more robust than its counter.

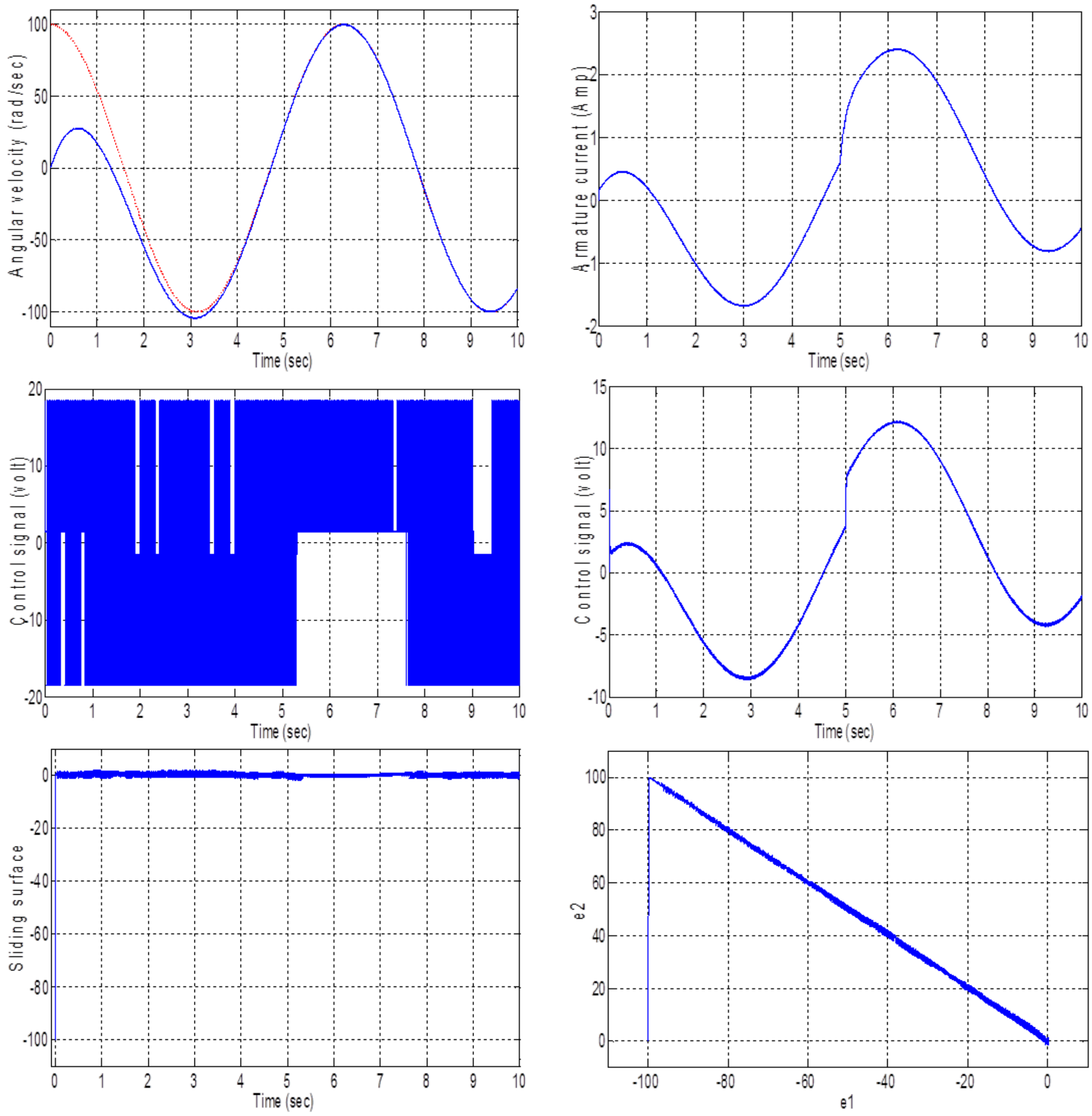

Figure (4) Simulation of different system responses due to sinusoidal input with disturbance
(a) Velocity response (b) Current behavior (c) Non-filtered actuating signal
(d) Filtered actuating signal & sliding surface behavior (f) phase plane of $e_2 - e_1$

7 References

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تصميم مسيطر انزلاقي تكاملي لمحرك تيار مستمر مؤازر

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الخلاصة:

يتميز المحرك المؤازر يسهولة التركيب والسيطرة وله تطبيقات عديدة. مع ذلك، فان هناك حتمية في ظهور او وجود (Uncertainties) بسبب تغير معلماته. لذلك يتوجب استخدام مسيطر متين للحفاظ على المتتطلبات المحددة والموصوفة للمنظومة بغض النظر عن تغير تلك المعلمات.

في هذا البحث تم اقتراح مسيطرين ذات النمط الانزلاقي لغرض السيطرة على سرعة المحرك ذو التيار المستمر تحت تاثير احمال مختلفة وهما: المسيطر التقليدي ذات النمط الانزلاقي وكذلك المسيطر التكاملي ذات النمط الانزلاقي.

اظهرت النتائج بان المسيطر التمامل ذات النمط الانزلاقي اعطى اداء اعلى وخصائص متابعة افضل من المسيطر التقليدي وتمكن من اجراء التعويض الناتج من تغير الحاصل في متغيرات المنظمومة بشكل افضل من نظيره.