Robust Controller Design for Two Wheeled Inverted Pendulum System

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Abstract

In this paper, the design of a robust controller for two wheeled inverted pendulum (TWIP) system is presented. In the first stage of the design, a full state feedback H_2 control is designed for stabilizing the inclination of (TWIP) system to upright position. The H_∞ controller for the stabilized system is synthesized in the second stage. The mathematical model of the system based on the Newtonian approach is developed. The results verify that the proposed controller can compensate the system parameter uncertainty with a more desirable time response specifications.

Keywords: Two wheeled inverted pendulum, H_2 full state feedback control, H_∞ controller, Robust control, Optimal control.

1. Introduction

Inverted pendulum is the subject of several studies in automatic control as an exemplary representative of a class of high-order nonlinear and non-minimum phase systems. In the recent years the two wheeled inverted pendulum became a very important system in industry, since it provides a large degree of flexibility and efficiency with respect to transportation, inspection, and operation [1]. Several practical systems have been executed according to TWIP models, such as the JOE, the Nbot, the Legway, and the Segway etc. Control of inverted pendulum is very challenging since one input has to control two variables which are pendulum angle and system position. The inverted pendulum system moves in x and y-plane (two degrees-of-freedom (DOF)), and it is a challenge to replace the fixed track of the inverted pendulum with two wheels which add an another degree-of-freedom (DOF) to allow motion on the plane. Those forms the two wheeled inverted pendulum system whose structure is a collection of two systems, a vehicle and an inverted pendulum. Control of the TWIP system becomes more complex after adding the heading angle to be controlled. Thus, for successful control of the system of two wheeled inverted pendulum, three variables, namely a pendulum angle, heading angle and a system position should be controlled properly by input voltage to push the wheels [2]. These reasons lead to increase the effort required towards the control design that ensures stability and robustness for two wheeled inverted pendulum system. Accurate model of a system is desired for controllers design especially if the controllers are model based controllers [3].

On the other hand, several researches on modeling and control of the two wheeled inverted pendulum have been introduced. Euler Lagrange methods were used in [4], Newton-Euler equations of motion method was implemented in [5]. Further, control technologies for controlling the two wheeled inverted pendulum has been studied. For instance, Linear Matrix Inequality (LMI) [6], Sliding Mode Control (SMC)[7], Linear Quadratic Gaussian (LQG) [1], Linear Quadratic Regulator Controller (LQR) [8], Adaptive Sliding Mode Control[9], Fuzzy Control [10] and Neural Network Based Motion Control [11].

In this work, the two wheeled inverted pendulum model is developed using Euler equations of motion. The robust state feedback H_2 control is applied to stabilize the system. To compensate the system parameters variations and to ensure the tracking, the H_∞ controller is designed.

2. System Modeling

The system consists of two wheels, chassis, DC motors, inverted pendulum, and control unit. The couple of wheels are supported by the chassis and the inverted pendulum. The wheels of the system are rotated by the DC motors with respect to the chassis. The DC motors will be controlled by the control unit to move the system and stabilize the inverted pendulum [12]. Figure 1 shows the schematic diagram of the transformation of the electrical energy from the DC power supply into the mechanical energy supplied to the load.

Based on Newton's 2nd law and Kirchhoff's voltage law the subsequent dominant equations can be developed as [7, 12]:

$$J\ddot{\theta}(t) - b\dot{\theta}(t) = K_m i(t) = T(t) \quad (1)$$

$$L\frac{di(t)}{dt} + Ri(t) = V(t) - K_e\dot{\theta}(t) \quad (2)$$

where J is the inertia moment of wheel with rotor around the wheel axis $(Kg.m^2)$, $\theta(t)$ is the rotation angle of the wheel (rad), b is the motor viscous friction constant $(N.m.sec.\backslash rad)$, K_m is the back electromotive voltage coefficient $(v.sec./rad)$, K_e is the magnetic torque coefficient (Nm/A), $T(t)$ is the magnetic torque of the rotor (Nm), L is the nominal rotor inductance (H), $i(t)$ is the armature current (A), R is the nominal rotor resistance (ohm) and $V(t)$ is the motor voltage (v).

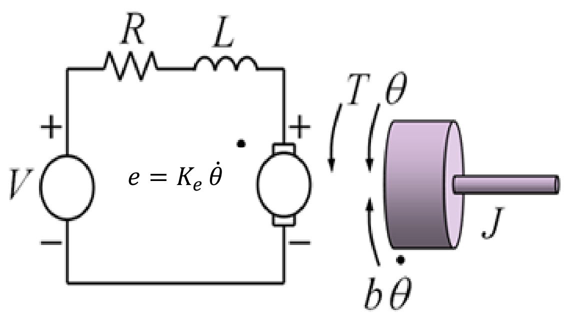


Figure 1: The diagram of DC motor [7].

Substituting equation (2) in equations (1), with neglecting the inductance and friction of the motor yields:

$$\ddot{\theta} = \frac{K_m}{\mathrm{RJ}}V - \frac{K_m K_e}{\mathrm{RJ}}\dot{\theta} \tag{3}$$

The torque of motor is:

$$T(t) = \frac{K_m V(t)}{R} - \frac{K_m K_e}{R}\ \dot{\theta}(t) \tag{4}$$

Based on balancing forces and moments on the wheels, chassis, and inverted pendulum, the motion equations of (TWIP) are developed [12]. Figure 2 shows the diagram of forces and moments affecting the (TWIP) system.

Balancing forces and moments acting on the left wheel are [7]:

$$M_w \ddot{x}(t) = H_{fL}(t) - H_L \tag{5}$$

$$I_w \ddot{\theta}_{LW}(t) = T_L(t) - H_{fL}(t) r \tag{6}$$

where M_w represents the wheel mass with DC motor (Kg), $x(t)$ represents the wheel position (m), $H_{fL}(t)$ represents the friction force between the left wheel and the ground (N), H_L represents the reaction force between left wheel and the inverted pendulum (N), I_w represents the wheel moment of inertia with rotor around the wheel axis $(Kg.m^2)$, $\theta_{LW}(t)$ represents the angle of the left wheel (rad), $T_L(t)$ represents the torque on the left wheel (Nm) and r represents the wheel radius (m). Regarding the right wheel, the equations are:

$$M_w \ddot{x}(t) = H_{fR}(t) - H_R \tag{7}$$

$$I_w \ddot{\theta}_{RW}(t) = T_R(t) - H_{fR}(t)\, r \tag{8}$$

where $H_{fR}(t)$ is the friction force between the right wheel and the ground (N), H_R is the reaction force between right wheel and the inverted pendulum (N), $\theta_{RW}(t)$ is the angle of the right wheel (rad) and $T_R(t)$ is the torque acting on the right wheel (Nm).

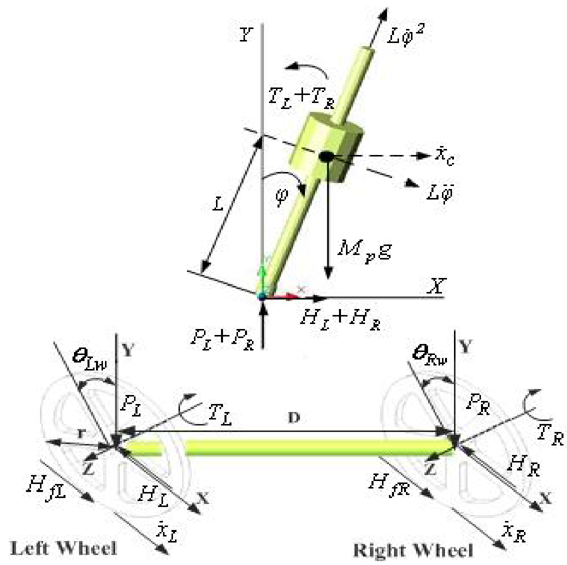


Figure 2: Free body of motion for two wheeled inverted pendulum [7].

For the inverted pendulum the equations are [7]:

$$(H_L + H_R) - M_p L\ddot{\emptyset}(t)\cos\emptyset(t) + M_p L\dot{\emptyset}^2(t)\cos\emptyset(t) = M_p \ddot{x}(t) \tag{9}$$

$$(H_L + H_R)cos\emptyset(t) + (P_L + P_R)sin\emptyset(t) - M_p g sin\emptyset(t) - M_p L\ddot{\emptyset}(t) = M_p L\ddot{x}(t)\cos\emptyset(t) \tag{10}$$

$$(H_L + H_R)L\,cos\emptyset(t) + (P_L + P_R)L\,sin\emptyset(t) + (T_L(t) + T_R(t)) = I_p\ddot{\emptyset}(t) \tag{11}$$

where $\emptyset(t)$ represents the inverted pendulum angle (rad), P_R, P_L represents the reaction force between left and right wheel and the inverted pendulum (N), M_p represents the pendulum mass (Kg), I_p represents the inverted pendulum moment of inertia around the wheel axis $(Kg.m^2)$ and g represents the gravitation acceleration (m/s^2).

The chassis and pendulum equations about the z-axis is [7]:

$$\ddot{\delta}(t)\, I_\delta = (H_L - H_R)\,\frac{D}{2} \qquad (12)$$

where $\delta(t)$ represents the rotation angle in y-axis (rad), I_δ represents the chassis moment of inertia with respect to y-axis (Kg.m^2) and D represents the lateral distance between two coaxial wheels (m).

From equations (1) to (12), the equations of motion of the (TWIP) system are:

$$\ddot{x}(t) = 2K_m\left(\frac{\gamma - Mp\,Lr}{Rr\beta}\right)V(t) + 2K_mK_e\left(\frac{\mathrm{M_p L}\,r-\gamma}{Rr^2\beta}\right)\dot{x}(t) + \frac{M_p^2L^2g}{\beta}\emptyset(t) \qquad (13)$$

$$\ddot{\emptyset}(t) = 2K_m\left(\frac{Mp\,L-\alpha r}{Rr\beta}\right)V(t) + 2K_mK_e\left(\frac{\alpha r-\mathrm{M_p L}}{\beta Rr^2}\right)\dot{x}(t) + \frac{Mp\,L\,g\alpha}{\beta}\emptyset(t) \qquad (14)$$

$$\ddot{\delta}(t) = \frac{DrK_m}{R\mu}\,(V_L(t) - V_R(t)) \qquad (15)$$

where

$$\beta = I_p\alpha + 2M_pL^2\left(M_w + \frac{I_w}{r^2}\right)$$

$$\alpha = \left(M_p + 2M_w + \frac{2I_w}{r^2}\right)$$

$$\gamma = \left(\mathrm{I_p} + M_pL^2\right)$$

The linearization of equations (13) to (15) can be done by assuming $\emptyset(t) = \pi + \theta(t)$, where $\theta(t)$ is the small angle from the vertical direction. That is $cos\emptyset = -1$, $\sin\emptyset = -\emptyset$ and $\left(\frac{d\emptyset}{dt}\right)^2 = 0 = \dot{\emptyset}^2$.

It is assumed that, the slip between the wheels and ground is neglected and the wheels always stay in contact with the ground. That is, there will be no movement in the z-axis and no rotation about the x-axis.

The resulting linearized model is:

$$\begin{bmatrix}\dot{x}(t)\\ \ddot{x}(t)\\ \dot{\emptyset}(t)\\ \ddot{\emptyset}(t)\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & a_{22} & a_{23} & 0\\ 0 & 0 & 1 & 0\\ 0 & a_{42} & a_{43} & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\\ \emptyset(t)\\ \dot{\emptyset}(t)\end{bmatrix} + \begin{bmatrix}0\\ b_{21}\\ 0\\ b_{41}\end{bmatrix}\mathrm{u(t)}$$

$$y(t) = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\\ \emptyset(t)\\ \dot{\emptyset}(t)\end{bmatrix} \qquad (16)$$

Where:

$$a_{22} = 2K_mK_e\left(\frac{\mathrm{M_p L}\,r-\gamma}{Rr^2\beta}\right)$$

$$a_{23} = \frac{M_p^2L^2g}{\beta}$$

$$a_{42} = 2K_mK_e\left(\frac{\alpha r-\mathrm{M_p L}}{\beta Rr^2}\right)$$

$$a_{43} = \frac{Mp\,L\,g\alpha}{\beta}$$

$$b_{21} = 2K_m\left(\frac{\gamma - 2Mp\,Lr}{Rr\beta}\right)$$

$$b_{41} = 2K_m\left(\frac{Mp\,L-\alpha r}{Rr\beta}\right)$$

The parameters of the (TWIP) are given in Table 1.

Table 1: System parameters [7].

Parameter	Value	Unit
r	0.05	m
Mp	1.13	Kg
I_p	0.004	$Kg.m^2$
I_w	0.001	$Kg.m^2$
L	0.07	m
K_m	0.006	$N.m/A$
R	3	ohm
Mp	0.03	K_g
K_e	0.007	K_g
g	9.81	m/s^2

3. Controller Design

In this work two control approaches are designed to control the two wheeled inverted pendulum system. The first approach is the full state feedback $\mathrm{H_2}$ control which is used to stabilize the system. The second approach is the $\mathrm{H_\infty}$ control which is designed for achieving the tracking and robustness for the system.

3.1. Full state feedback $\mathrm{H_2}$ stabilizing controller

Consider the full state $\mathrm{H_2}$ control block shown in Figure 3 and assume that

$$M = \begin{bmatrix}A & B_1 & B_2\\ C_1 & 0 & D_{12}\\ I & 0 & 0\end{bmatrix} \qquad (17)$$

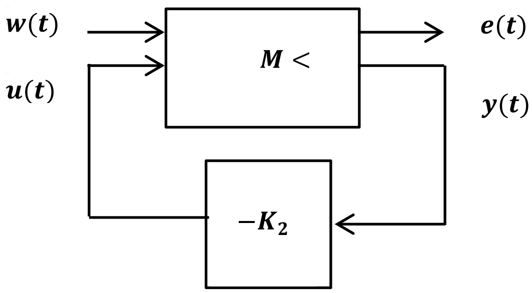


Figure 3: A full state feedback H_2 control [14].

where $w(t)$ represents the external inputs (set point, disturbance), $x(t)$ is the state vector, $y(t)$ is output, $u(t)$ is control action and $e(t)$ is the output to be minimized.

The following assumptions are made [14]:

1-(A, B_1) and (A, B_2) are stabilizable.

2- (C_1, A) is detectable.

The model can be expressed by:

$$\dot{x}(t) = Ax(t) + B_1\, d(t) + B_2\, u(t) \quad (18)$$

$$e(t) = C_1x(t) + D_{12}\, u(t) \quad (19)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix}, B_1 = 0,\ B_2 = \begin{bmatrix} 0 \\ b_{21} \\ 0 \\ b_{41} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} and\ D_{12} = 0 \quad (20)$$

$d(t)$ is the white noise vector .

The H_2 norm of the system error due to a white noise input is:

$$\|T_{ed}\|_{H_2}^2 = E(e^T(t)e(t)) \quad (21)$$

where T_{ed} is the transfer function from $d(t)$ to $e(t)$,then

$$e^T(t)e(t) = x(t)^T Q_f x(t) + 2x(t)^T N_f u(t) + u(t)^T R_f\, u(t) \quad (22)$$

$$Q_f = C_1{}^T C_1\ ,\ N_f = C_1{}^T D_{12}\ , R_f = D_{12}{}^T D_{12}.$$

Consequently, the cost function to be minimized is:

$$J = \int_{t_0}^{t_f} [\, x(t)^T Q_f x(t) + 2x(t)^T N_f u(t) + u(t)^T R_f\, u(t)]dt \quad (23)$$

The optimal control action is:

$$u(t) = -K\, x(t) \quad (24)$$

where

$$K = R_f{}^{-1}(B_1{}^T P +\ N_f{}^T) \quad (25)$$

where P represents the symmetric and positive definite transformation matrix and it can be obtained by the following Riccati equation:

$$\left(A - B_2 R_f{}^{-1}\, N_f{}^T\right)^T P +\ P(A - B_2 R_f{}^{-1}\, N_f{}^T) - PB_2\, R_f{}^{-1} B_1{}^T P +\ Q_f - N_f\, R_f{}^{-1} N_f{}^T = 0 \quad (26)$$

Figure 4 shows the block diagram of the system with the stabilizing H_2 controller.

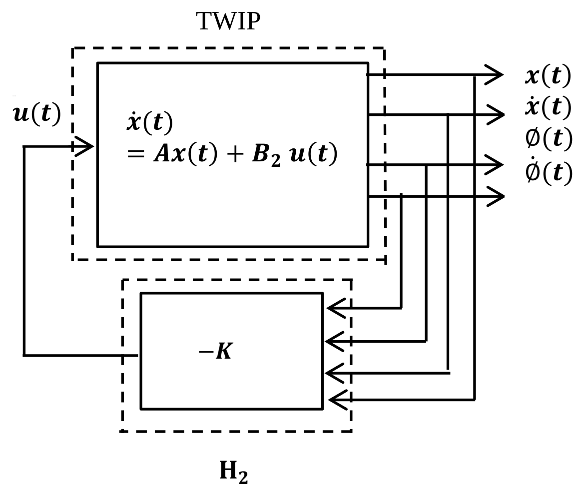


Figure 4: Block diagram of the system with the stabilizing H_2 controller.

3.2. H_∞ controller synthesis

For robust control, the H_∞ control is considered one of the most known techniques available nowadays. It is a method in control theory for the design of optimal controllers. The H_∞ control is characterized an effective method for rejecting disturbances and noise that appear in the system. It has proven to be one of the best techniques in linear control system design. The performance analysis is defined in terms of sensitivity function $S(s)$, complementary sensitivity function $T(s)$, and control sensitivity function $KS(s)$ as in the following equations [13]:

$$S(s) = (1 + G(s)K(s))^{-1} \quad (27)$$

$$T(s) = G(s)K(s)(1 + G(s)K(s))^{-1} \quad (28)$$

$$KS(s) = K(s)(1 + G(s)K(s))^{-1} \quad (29)$$

By choosing various frequency ranges, the H_∞ norms can be determined. These ranges for different optimizations can be indicated as three different weighting functions, W_1 (performance), W_2(control) and W_3(uncertainty) such that the infinite norm of the mixed sensitivity functions is minimized as:

$$\left\| \begin{matrix} W_1(jw)S(jw) \\ W_2(jw)KS(jw) \\ W_3(jw)T(jw) \end{matrix} \right\|_\infty < \gamma \qquad (30)$$

where γ is a positive integer number. If $\gamma \leq 1$, the robust stability and performance are achieved. The standard H_∞ control problem is shown in Figure 5, where $N(s)$ is the augmented plant model, the set of input $\{w(s), u(s)\}$ to the set of output $\{e(s), y(s)\}$. The vector $w(s)$ represents the exogenous inputs external to the closed loop system, the vector $e(s)$ is the minimized output vector, $u(s)$ is the control input, $y(s)$ is the output, and $K(s)$ is the desired optimal controller [13].

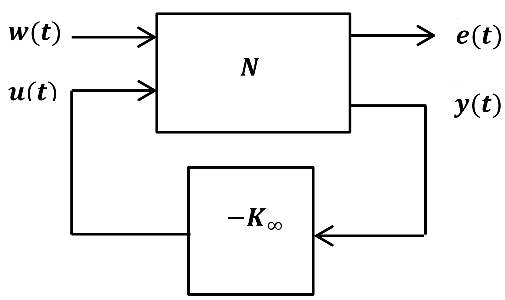


Figure 5: Standard H_∞ Control Problem [13].

Then the system transfer function matrix is partitioned as [13]:

$$\begin{bmatrix} e(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} N_{ew}(s) & N_{eu}(s) \\ N_{yw}(s) & N_{yu}(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \qquad (31)$$

Where $w(s)$ represents the exogenous inputs vector, $e(s)$ represents the outputs to be minimized vector and $u(s)$ represents the control signal.

By substituting the control law, $u(s) = K(s)y(s)$, into equation (31), the relationship between the output to be minimized and the external inputs is obtained as:

$$e(s) = [N_{ew}(s) + N_{eu}(s)K(s)\{I - N_{yu}(s)K(s)\}^{-1} N_{yw}(s)]w(s) = [F_l(N, K)]w(s) \qquad (32)$$

The H_∞ optimal control design consists of finding a stabilizing controller $K(s)$, such that the H_∞ norm of the closed loop transfer matrix $F_l(N, K)$ is minimized, *i.e.*

$$\|F_l(N, K)\|_\infty < \gamma \qquad (33)$$

It is important to refer that the design of H_∞ controller is done using Matlab environment and especially using the order (mixsyn). The design of H_∞ controller is applied to the plant stabilized by the full state feedback H_2 control explained in section (3.1). Figure 6 shows the block diagram of the augmented plant including the stabilized plant with the weighting functions. It shows that the mixed sensitivity case is used to design the H_∞ controller for achieving a desirable robust performance.

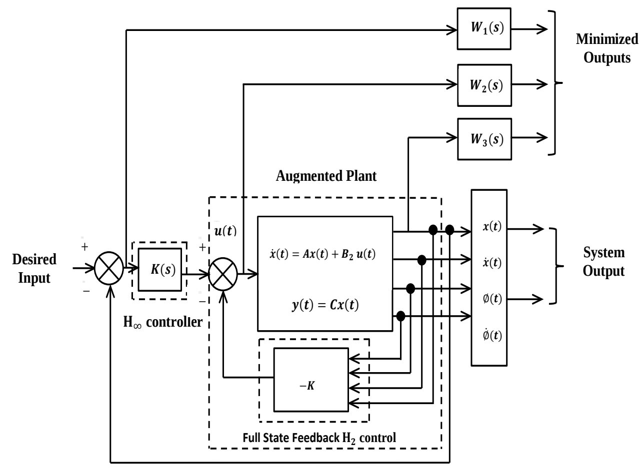


Figure 6: Block diagram of the stabilized system with H_∞ controller.

4. Results and Discussion

The considered two wheeled inverted pendulum system has the following eigenvalues $\{0, -0.0056, -11.011, 11.012\}$ which means that the system is unstable because it has a root on the right hand plane. To stabilize the system a full state feedback H_2 controller is applied. Figure 7 shows the time response specifications of the (TWIP) system with H_2 stabilizing controller. It shows that the controller can stabilize the system within 3 $seconds$ and the pendulum angle oscillates between -0.201 to 0.065 degree. The resulting state feedback gains are:

$K = [-57.988 - 67.015 - 2310.061 - 207.335]$ and the new closed loop eigenvalues are: $\{-11.050 \;\; -10.969 \;\; -1.125 + 1.025i \;\; -1.125 - 1.025i\}$ which means that the system is stable. The weighting matrices that are required to design the controller are selected as:

$$Q_f = \begin{bmatrix} 119546 & 0 & 0 & 0 \\ 0 & 9900.8 & 0 & 0 \\ 0 & 0 & 0.1950 & 0 \\ 0 & 0 & 0 & 0.0453 \end{bmatrix}, \; N_f = \begin{bmatrix} 0.9092 \\ 0.09545 \\ 0.9364 \\ 0.7179 \end{bmatrix}, \; R_f = [35.5519].$$

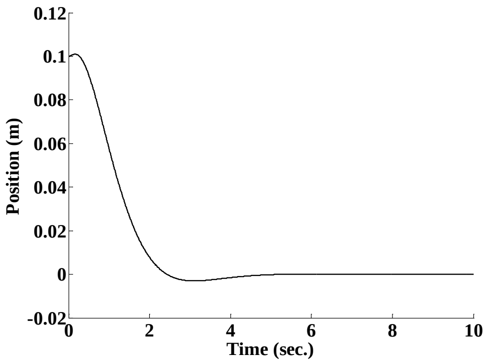
Moreover, the resulting control signal is within the range of system input voltage.

On the other hand, to achieve the required robustness for the (TWIP) system in the presence of system parameter uncertainty, the H_∞ controller is designed. The weighting functions are selected by trial and error as:

$$W_1(s) = \frac{0.0999\,s^2 + 0.99\,s + 0.25}{0.7006\,s^2 + 1.044\,s + 2.36} \quad (34)$$

$$W_2(s) = \frac{0.2805\,s + 0.1536}{0.1104\,s + 4.194} \quad (35)$$

$$W_3(s) = \frac{0.103\,s^2 + 0.4988\,s + 0.5596}{0.0115\,s^2 + 0.5047\,s + 1.5} \quad (36)$$



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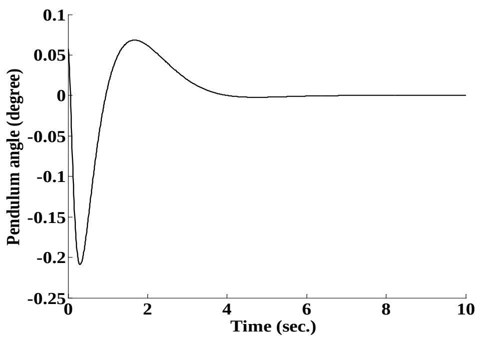

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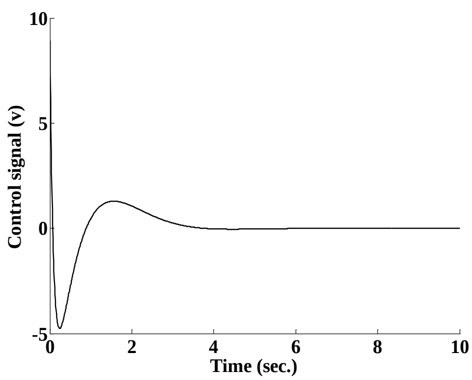


(c)

Figure 7: Time response specifications of the system with full state feedback H_2 stabilizing controller, $x_0 = [0.1\ 0\ 0.001\ 0]$, a) system position, b) pendulum angle, c) control signal.

The H_∞ control minimizes the cost function in equation (33) using γ iteration. The resulting γ is 0.9495, that is the robustness condition has been satisfied. The required position and pendulum angle are (0.25 m) and (0 ^{o}).

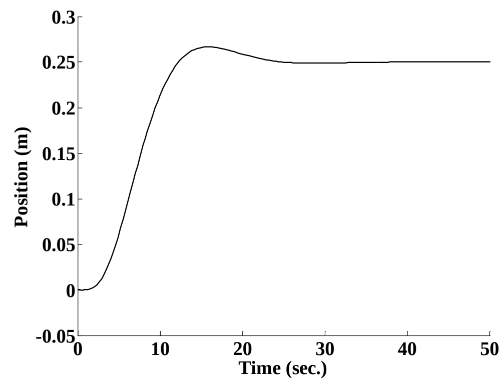
The resulting H_∞ controller is:

$$K(s) = \frac{s^3 + 53\,s^2 - 0.314\,s - 105}{s^4 + 5.406\,s^3 + 70.39\,s^2 + 61.66\,s} \quad (37)$$

Figure 8 shows the time response specifications of the system with H_2 stabilizing controller and H_∞ controller. The obtained time response specifications are: $t_r = 10\ seconds, t_s = 18\ seconds$ and , $M_P = 6.4\ \%$ and the pendulum angle oscillates between 0.0143 to -0.006 degree. Moreover, to test the robustness of the controlled system a variation of $\pm 10\%$ in system parameters is considered. It is shown that the designed controller can compensate the system uncertainty as shown in Figure 9. It is obvious that a low control action has been achieved.

5. Conclusion

The design of full state feedback H_2 controller for stabilizing the two wheeled inverted pendulum system has been presented. The H_∞ controller was applied to the stabilized system to achieve a more desirable robust performance. The results showed that the proposed controller can effectively stabilize the system and compensate the variation in system parameters. Moreover, it was shown that the proposed controller has achieved a minimum deviation in pendulum angle.



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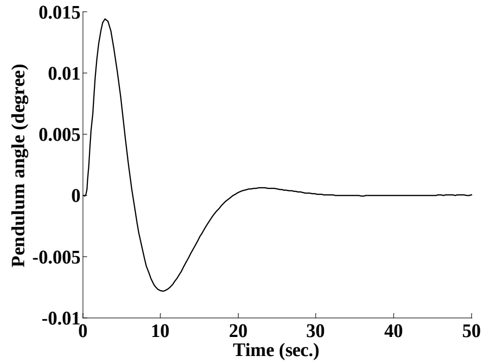

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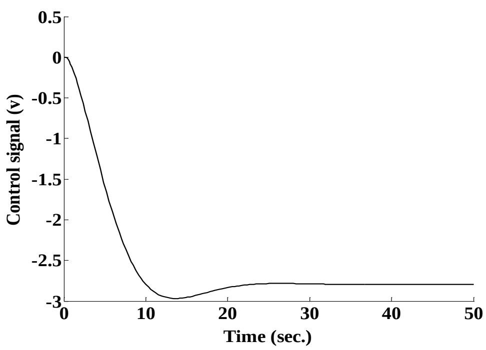
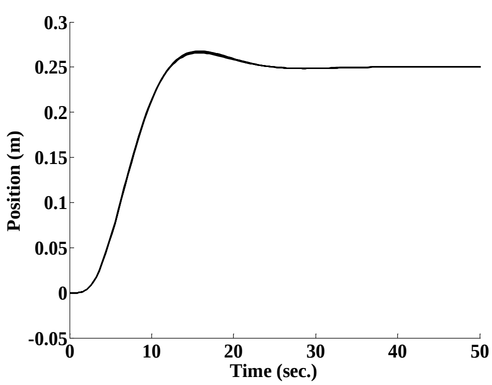


(c)

Figure 8: Time response specifications of the system using full state feedback H_2 stabilizing controller and H_∞ controller, a) system position, b) pendulum angle, c) control signal.



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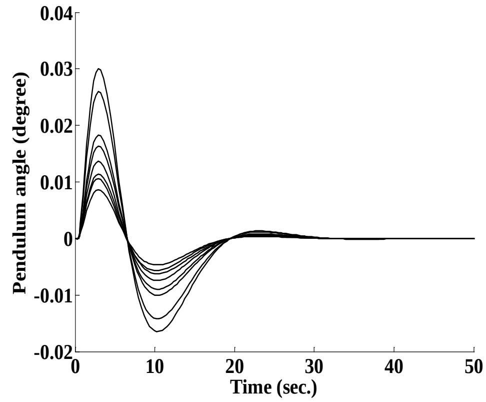

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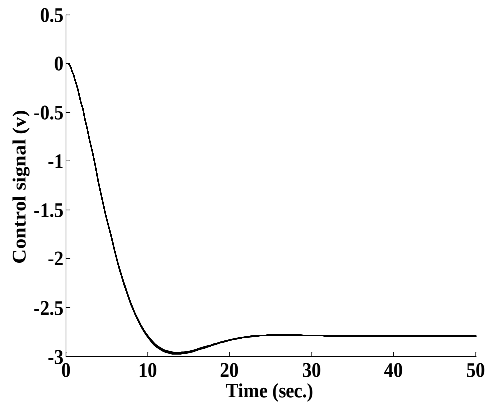

(c)

Figure 9: Time response specifications of the uncertain system using full state feedback H_2 stabilizing controller and H_∞ controller, a) system position, b) pendulum angle, c) control signal.

6. Reference

[1] R. Eide, P. M. Egelid, A. Stamsø and H. R. Karimi, "LQG Control Design for Balancing an Inverted Pendulum Mobile Robot", *Intelligent Control and Automation*, Vol. 2, No. 2, pp. 160-166, 2011.

[2] G. H. Lee, S. Jung, "Line Tracking Control of a Two-Wheeled Mobile Robot Using Visual Feedback", *International Journal of Advanced Robotic Systems*, Vol. 10, No. 3, pp. 1-8, 2013.

[3] A. Calanca, L. M. Capisani, A. Ferrara and L. Magnani, "MIMO Closed Loop Identification of an Industrial Robot", *IEEE Transactions on Control Systems Technology*, Vol. 19, No. 5, pp. 1214-1224, 2011.

[4] K. M. Goher, M. O. Tokhi, and N. H. Siddique, "A Comparison of Controllers for Balancing Two Wheeled Inverted Pendulum Robot", *International Journal of Mechanical & Mechatronics Engineering*, Vol. 14, No. 3, pp. 62-68, 2014.

[5] K. M. Goher, M. O. Tokhi and N. H. Siddique, "Dynamic Modeling and Control of a Two Wheeled Robotic Vehicle with Virtual Payload", *ARPN Journal of Engineering and Applied Sciences*, Vol. 6, No. 3, pp. 7-41, 2011.
[6] S. Dhobaley, P. Bhopale and A. Pandey, "LMI Based Control for Balancing an Inverted Pendulum Mobile Robot", *International Journal of Engineering Research & Technology*, Vol. 3, No. 4, pp. 2023-2026, 2014.
[7] N. T. Phuong, H. D. Loc and T. Q. Thuan, "Control of Two Wheeled Inverted Pendulum Using Sliding Mode Technique", *International Journal of Engineering Research and Applications*, Vol. 3, No. 3, pp. 1276-1282, 2013.
[8] A. N. K. Nasir, M. A. Ahmad and R. M. T. Raja Ismail, "The Control of a Highly Nonlinear Two-wheel Balancing Robot: A Comparative Assessment between LQR and PID-PID Control Schemes", *International Scholarly and Scientific Research & Innovation*, Vol. 4, No. 10, pp. 176-181, 2010.
[9] M. Yue, X. Wei and Z. Li, "Adaptive Sliding Mode Control for Two-Wheeled Inverted Pendulum Vehicle Based on Zero Dynamics Theory", *Nonlinear Dynamics*, Vol. 76, No. 1, pp. 459-471, 2013.
[10] C. H. Huang, W. J. Wang and C. H. Chiu, "Design and Implementation of Fuzzy Control on a Two-Wheel Inverted Pendulum", *IEEE Transactions on Industrial Electronics*, Vol. 58, No. 7, pp. 2988-3001, 2011.
[11] C. Yang, Z. Li, R. Cui and B. Xu, "Neural Network-Based Motion Control of an Under-actuated Wheeled Inverted Pendulum Model", *IEEE Transactions on Neural and Learning Systems*, Vol. 25, No. 11, pp. 2004-2016, 2014.
[12] K. D. Do and G. Seet, "Motion Control of a Two Wheeled Mobile Vehicle with an Inverted Pendulum", *Journal of Intelligent & Robotic Systems*, Vol. 60, No. 3, pp. 577-605, 2010.
[13] A. Bansal and V. Sharma, "Design and Analysis of Robust H-infinity Controller", *Control Theory and Informatics*, Vol.3, No. 2, pp. 7-14, 2013.
[14] C. Fauvel, F. Claveau and P. Chevrel, "A Generically Well-Posed H_2 Control Problem for a One Shot Feedforward and Feedback Synthesis", *17th International Conference System Theory, Control and Computing (ICSTCC)*, *IEEE*, pp. 201 - 206, 2013.

تصميم مسيطر رصين لنظام بندول مقلوب ذي عجلتين

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الخلاصة

في هذا البحث، تم تصميم مسيطر رصين لنظام بندول مقلوب ذي عجلتين. في المرحلة الاولى تم تصميم مسيطر ذو تغذية رجعية لكل متغيرات النظام باستخدام طريقة ال H_2 من اجل استقرارية ميل نظام البندول المقلوب عن الوضع العمودي. في المرحلة الثانية تم تصميم مسيطر ال H_∞ للنظام المستقر. تم اعتماد النموذج الرياضي للنظام والممثل بطريقة نيوتن. النتائج تبين ان المسيطر المقترح بامكانة تعويض التغييرات التي تحدث في معاملات النظام وبمواصفات استجابة زمنية مرغوب فيها.

الكلمات المفتاحية: بندول مقلوب ذي عجلتين ، مسيطر H_2 ، مسيطر H_∞ ،سيطرة رصينة ،سيطرة مثلى.