Divergence and Parametric Instability of Belts by using Bolotin Method

Mahmud R. Ismail

mahmech2001@yahoo.com

Alnahrain University - College of Engineering - Mechanical Engineering Dep.

Abstract:

Belt is a traveling continuous system.Such a system can subjected to a static divergence and parametric instability. This depend on whether the system parameters are constant or varying with the time ,respectively .In this paper ,instability problem is solved analytically .Bolotin method is used to evaluate the boundaries which separate the stable and instable regions.The present solution is checked with another solution available in the literature where the Variation principle is used .The results showed a good agreement where the maximum error do not exceed 5%. The effect of belt tension and transmitting speed on stability and natural frequency are studied .The results show that increasing belt tension can improve both buckling and parametric instability. Whilst, increasing the speed or its mean value is limited to critical values to avoid buckling or parametric instability, respectively.

Keywords: Bolotin method, parametric instability, divergence instability, initial tension1.

Introduction

Instability problem of translating media covering band-saws, paper webs, transmission belts,.etc, has been of academic and engineering interest.

Belts are traveling continuous systems. Such systems are modeled as axially moving strings or beams with simply supported end conditions [1].

When a belt is working at high speeds or subjected to fluctuated speeds or tension arising from the influence of variable loads. two kinds of instabilities can be occurred ;The first kind is the static divergence instability or buckling instability .This is associated with the constant translating velocity or tension, The dynamical behavior of such translating media is governed by a partial differential equation with constant coefficients :The second kind is the parametric instability .This kind occurs when the translating velocity or tension has a time dependent periodic component superposed on its mean .In this case the coefficients of the partial differential equation will become time-dependent periodic or Mathieu equation .Such cases can be noticed in many practical applications such as in cutting tools machines, compressors and industrial machines .

The major task for safe operation is to determine the limiting values of system parameters at which buckling occur and to separate stability and instability regions of parametric instability.

Parametric instability of axially moving media with a periodic tension has been investigated by some researchers. Wickert and Mote [2] reviewed the dynamical behavior of moving materials with a focus on moving strings and beams. Mote [3] evaluated the parametric instability boundary of a translating string using numerical methods that involve the replacement of the spatial derivatives with finite differences and integration of the set of Mathieu equations. Pellicano and Vestroni [4] studied the complex eigenvalue of high speed moving string and tested stability under parametricexcitation for periodic loading. Mockensturm et al. [5] addressed the issue of stability and limit cycles of moving strings parametrically excited. Pakdemirli et al.[6] investigated the stability of an axially accelerating string. Recently, Abrate [7] studied the parametric instability of axially moving media subjected to multi-frequency tension and speed fluctuations.

The available methods for the evaluation of parametric instability boundaries of such problems are theperturbation, variational and Galerkin method [8].The perturbation and variational method can predict a portion of stability boundaries. Galerkin method is based on reducing the partial differential equation with periodic coefficients into a set of ordinary differential equations with periodic coefficients, the resulting ordinary differential equations can be either cast into an eigenvalue problem or solved using the perturbation methods .Galerkin method requires trial functions satisfying both essential boundary conditions and natural boundary conditions.

Liu and Huang[9] had investigated the problem using a variational method . The parametric instability problem was reduced into a stationary value problem in the form of a classical Rayleigh quotient. As a result, the trial functions do not have to satisfy the natural boundary conditions.

A linear analysis has been widely applied to determine the onset of parametric instability in a belt. The primary parametric instability and the fundamental summation resonance have been given special attention.

Besides to the linear analysis nonlinear analysis is also existed in the literature The effect of nonlinearities have long been recognized to play a significant role in the parametrically excited .The governing nonlinear partial differential equation was solved numerically .Numerical results indicate that the effects of different system parameters (like band speed, initial tension) are seen to be very similar to those exhibited by a Duffing oscillator [10].In These solutions instability is compared with the ‘limit cycles’ response instead of the equilibrium condition as in the linear analysis. The existence of limit cycles in a traveling string has been reported recently by Chakraborty et al.[11]. In this reference, the equation of motion has been discretized to a set of coupled, non-linear ordinary differential equations with time-varying coefficients or scale parameter. The complex linear eigenfunctions were used as a biases function to discrete the continues system to finite degree of freedom. The effects of both external and parametric excitations on a Duffing oscillator have also been studied by Chakraborty and. Mallik [12]. They showed that the nonlinear response is normally periodic.

The concept of non-linear normal modes has been used to obtain the near resonance response under a harmonic excitation [13]. Since the stationary normal modes do not exist for a traveling continuous system, In the present work, the linear problem of stability for both static and parametric instability was solved analytically. Bolotin method is used to separate the primary regions of parametric instability .The basic concept of the solution is to predict the limiting cycle solution in term of the amplitudes and frequencies of the periodic component which gives the boundaries between stability and instability.

Theoretical consideration

Consider a belt running between two pulleys as shown in Fig. 1.For an element of a belt of mass (*dm*) is moving at velocity of V and vibrates in y direction ,the resultant lateral velocity u is (shown in Fig. 2),[8] ;

$$u=\frac{\partial y}{\partial t}+V\frac{\partial y}{\partial x} \qquad \text{........ (1)}$$

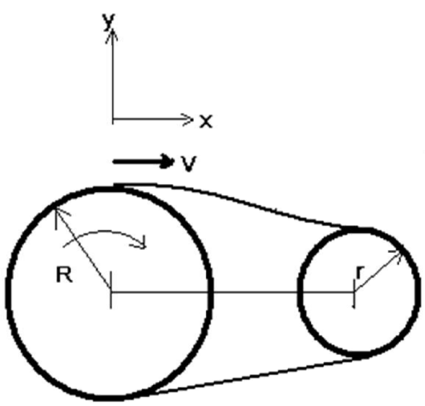


Fig.1: schematic diagram of moving belt

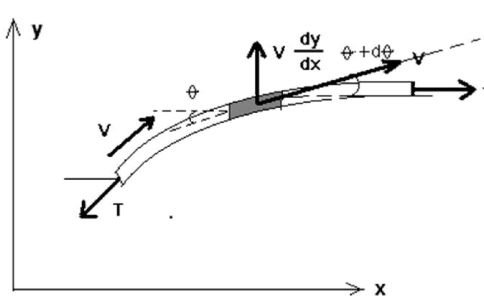


Fig.2:forces on belt element

The inertia force of the element is;

$$F=dm\frac{du}{dt}=dm\frac{d}{dt}\left(\frac{\partial y}{\partial t}+V\frac{\partial y}{\partial x}\right) \qquad \text{..... (2)}$$

Performing the differentiation giving;

$$F=dm\left(V^2\frac{\partial^2 y}{\partial x^2}+\frac{\partial y}{\partial x}\frac{\partial v}{\partial t}+2V\frac{\partial^2 y}{\partial x\partial t}+\frac{\partial^2 y}{\partial t^2}\right) \qquad \text{....(3)}$$

Considering the tension (T) in the belt as shown in Fig.2, summation of forces at y direction gives;

$$F=T(\theta+\frac{\partial \theta}{\partial x}dx)-T\theta=T\frac{\partial \theta}{\partial x}dx=T\frac{\partial^2 y}{\partial x^2}dx \qquad \text{..... (4)}$$

Substitute Eq.3 into 4 and noting that $dm=m\ dx$ produces the following equation of motion;

$$(mV^2-T)\frac{\partial^2 y}{\partial x^2}+2mV\frac{\partial^2 y}{\partial x\partial t}+m\frac{\partial V}{\partial t}\frac{\partial y}{\partial x}+m\frac{\partial^2 y}{\partial t^2}=0 \qquad \text{...... (5)}$$

Eq.5 can be put in the following dimensionless form;

$$(V^2-\gamma)\ \eta''+2V\dot{\eta}'+\dot{V}\eta'+\ddot{\eta}=0 \qquad \text{....... (6)}$$

Where;

$\eta = y/L, \zeta = x/L$ and $\gamma = T/m$

Natural Frequency

In this case, one must assume constant speed, so that Eq.6 reduces to;

$$(V^2-\gamma)\ \eta''+2V\dot{\eta}'+\ddot{\eta}=0 \qquad \text{.......(7)}$$

Let for harmonic motion, the solution is of the following form;

$$\eta(\zeta,t)=\varphi(\zeta)e^{i\Omega t} \qquad \text{...............(8)}$$

Substituting Eq.8 into 7 giving;

$$(V^2-\gamma)\varphi''+2iV\Omega\varphi'-\Omega^2\varphi=0 \qquad \text{...... (9)}$$

The second order differential equation can be solving by seeking the following solution;

$$\varphi(\zeta) = C_1 e^{i\lambda 1 \zeta L} + C_2 e^{i\lambda 2 \zeta L} \quad \text{........ (10)}$$

Where C_1 and C_2 are arbitrary constants, λ_1 and λ_2 are the roots of the polynomial equation;

$$(V^2 - \gamma)\lambda^2 + 2V\Omega\lambda + \Omega^2 = 0 \quad \text{...... (11)}$$

Substituting the values of the roots from Eq .11 into Eq.10 and making the use of Eq.8, the general solution becomes [4];

$$\eta(\zeta,t) = e^{\frac{-iV\Omega L\zeta}{(V^2-\gamma)}} (A\sin\alpha\zeta + B\cos\alpha\zeta) e^{i\Omega\tau} \quad \text{...... (12)}$$

Where;

$$\alpha = \Omega L \sqrt{\frac{\gamma}{(V^2-\gamma)^2}} \quad \text{...... (13)}$$

The boundary conditions of a belt portion traveling between two pulleys are :-

$\eta(0,\tau)=0$ and $\eta(1,\tau)=0$ ....(14)

Applying the boundary conditions on the general solution given in Eq.12 can give the following equation of the natural frequencies;

$$\Omega = \frac{n\pi}{L}\sqrt{\frac{(V^2-\gamma)^2}{\gamma}}, \; n=1, 2, 3, \ldots \quad (15)$$

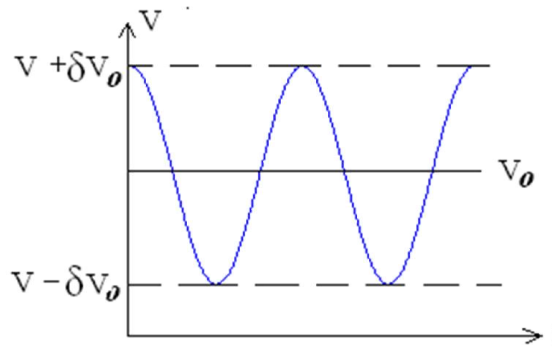
Static Divergence instability

Eq.15 shows that the natural frequencies can becomes zero .This means that the belt can be subjected to static divergence or buckling instability. This condition can be satisfied when;

$$V = \sqrt{\gamma} = \sqrt{T/m} \quad \text{....... (16)}$$

Parametric Instability

Consider a belt is subjected to a fluctuated speed with frequency Ω and variable amplitude of δ .The speed is fluctuated around its mean V_o.as shown in the figure below



From the above figure, the mathematical representation of such a speed is;

$$V=V_o\,(1+\delta\cos\Omega\tau) \quad \text{..........(17)}$$

Substituting Eq.17 into 6 gives;

$$[V_o^2(1+\delta\cos\Omega\tau)^2-\gamma]\eta''+2V_o(1+\delta\cos\Omega\tau)\dot{\eta}' - \delta V_o\Omega\sin\Omega\tau\,\eta' + \ddot{\eta} = 0 \quad \text{......... (18)}$$

Analytical solution for periodic differential equation like Eq.18 does not exist in literature, however there are many partial solutions can be employed to evaluate some interesting dynamical behaviors .In this regard, Bolotin's solution is an effective method to predict the periodic zone of the solution which lead to the boundaries instability regions.

In using Bolotin solution one can seek the following solution [14];

$$\eta(\zeta,\tau) = \sum_{k=1,3,5,\ldots}^{\infty} X_k(\zeta)\sin(k\Omega\tau/2) + Y_k(\zeta)\cos(k\Omega\tau/2) \quad \text{...... (19)}$$

Substituting this solution into Eq.18, resulting the following series;

$$\sum_{k=1,3,5,...}\Big[\left\{\left(V_o^2-\gamma+\frac{V_o^2\delta^2}{2}\right)\frac{dX_k}{d^2\zeta}-\left(\frac{k}{2}\right)^2\Omega^2X_k-V_o\Omega k\frac{dY_k}{d\zeta}\right\}\sin(\tfrac{1}{2}k\Omega\tau)$$

$$+\left\{\left(V_o^2-\gamma+\frac{V_o^2\delta^2}{2}\right)\frac{d^2Y_k}{d^2\zeta}-\left(\frac{k}{2}\right)^2\Omega^2Y_k+V_o\Omega k\frac{dX_k}{d\zeta}\right\}\cos(\tfrac{1}{2}k\Omega\tau)$$

$$+\left\{-\left(\frac{k\Omega}{2}\right)V_o\delta\frac{dY_k}{d\zeta}+V_o^2\delta\frac{d^2X_k}{d\zeta^2}-\tfrac{1}{2}V_o\Omega\frac{dY_k}{d\zeta}\right\}\sin\left(\frac{k+2}{2}\Omega\tau\right)$$

$$+\left\{\left(\frac{k\Omega}{2}\right)V_o\delta\frac{dX_k}{d\zeta}+V_o^2\delta\frac{d^2Y_k}{d\zeta^2}-\tfrac{1}{2}V_o\delta\Omega\frac{dX_k}{d\zeta}\right\}\cos\left(\frac{k+2}{2}\Omega\tau\right)$$

$$+\left\{-\left(\frac{k\Omega}{2}\right)V_o\delta\frac{dY_k}{d\zeta}+V_o^2\delta\frac{d^2X_k}{d\zeta^2}+\tfrac{1}{2}V_o\Omega\frac{dY_k}{d\zeta}\right\}\sin\left(\frac{k-2}{2}\Omega\tau\right)$$

$$+\left\{\left(\frac{k\Omega}{2}\right)V_o\delta\frac{dX_k}{d\zeta}+V_o^2\delta\frac{d^2Y_k}{d\zeta^2}-\tfrac{1}{2}V_o\delta\Omega\frac{dX_k}{d\zeta}\right\}\cos\left(\frac{k-2}{2}\Omega\tau\right)$$

$$+\frac{V_o^2\delta^2}{4}\frac{d^2X_k}{d\zeta^2}\left\{\sin\left(\frac{k+4}{2}\Omega\tau\right)+\sin\left(\frac{k-4}{2}\Omega\tau\right)\right\}$$

$$+\frac{V_o^2\delta^2}{4}\frac{d^2Y_k}{d\zeta^2}\left\{\cos\left(\frac{k+4}{2}\Omega\tau\right)+\cos\left(\frac{k-4}{2}\Omega\tau\right)\right\}\Big]=0 \quad \text{…….. (20)}$$

Bolotin has also showed that this series is rapidly converges, and when one term is taken (k=1) ,satisfactory results can be obtained .Doing so and separating the coefficients of sin($\Omega\tau/2$) and cos($\Omega\tau/2$) one can gets;

$$(V_o^2-\gamma+\delta^2V_o^2/2-\delta V_o^2)X_1''-\tfrac{1}{4}\Omega^2X_1-V_o\Omega Y'_1=0$$
$$(V_o^2-\gamma+\delta^2V_o^2/2+\delta V_o^2)Y_1''-\tfrac{1}{4}\Omega^2Y_2+V_o\Omega X'_1=0 \quad \text{……. (21)}$$

Eqs.21 are two coupled ordinary differential equations and their solutions give the upper and lower boundaries of principal instability regions.

To solve eqs.21 the following solution can be used [14];

$$X_1=\sum_{j=1}^{4}C_{1j}e^{\lambda j\zeta} \quad \text{and}$$

$$Y_1=\sum_{j=1}^{4}C_{2j}e^{\lambda j\zeta} \quad \text{…… (22)}$$

Where C_{1j} 's and C_{2j} 's are arbitrary constants. These constants are related to each other when they substituted into any of Eqs.21.

When the solutions given in Eqs.22 are applied to Eqs.21, the following determinant will be resulted:-

$$\begin{vmatrix}(V_o^2-\gamma+\tfrac{1}{2}\delta^2V_o^2-\delta V_o^2)\lambda^2-1/4\Omega^2 & -V_o\Omega\lambda\\ V_o\Omega\lambda & (V_o^2-\gamma+\tfrac{1}{2}V_o^2\delta^2+\delta V_o^2)\lambda^2-1/4\Omega^2\end{vmatrix}=0 \quad \text{…… (23)}$$

Eq.23 can be used to find the four roots of λ which are denoted by λ_j in Eq.22.

The boundary conditions are as follows;

$X_1(0)=0$, $X_1(1)=0$,
$Y_1(0)=0$, $Y_1(1)=0$ …….(24)

Applying the boundary conditions on the solutions given in Eq.22 gives a matrix equation .For a nontrivial solution the following determinant must be satisfied;

$$\begin{vmatrix}1 & 1 & 1 & 1\\ e^{\lambda 1} & e^{\lambda 2} & e^{\lambda 3} & e^{\lambda 4}\\ K1 & K2 & K3 & K4\\ K1e^{\lambda 1} & K2e^{\lambda 2} & K3e^{\lambda 3} & K4e^{\lambda 4}\end{vmatrix}=0 \quad \text{….. (25)}$$

Where:-

λ_1,…… λ_4 arethe roots of Eq.23 as stated before, and ,

$K_j=Vo\,\Omega\,\lambda_j/[(V_o^2-\gamma+\delta^2V_o^2/2-\delta V_o^2)\lambda_j^2-1/4\Omega^2]$, j =1 to 4

The values of (Ω) and (δ) which satisfying Eq.25 give the upper and lower boundaries of principal instability regions for the belt , A computer code was written in MATLAB software for solving Eq.25.

Results and Discussions

Shown in Fig.3, the effect of the two main belt parameters namely; the speed and tension on the fundamental natural frequency .It is clear from the figure that increasing the speed reduces the natural frequency and there is a possibility that the natural frequency to be zero as it is clear from examining Eq.15, hence,divergence instability can be initiated .For example this figure shows that at T=500N the natural frequency is nearly zero at V=40.

On the other hand, this figure indicates that increasing the tension increase the natural frequency.

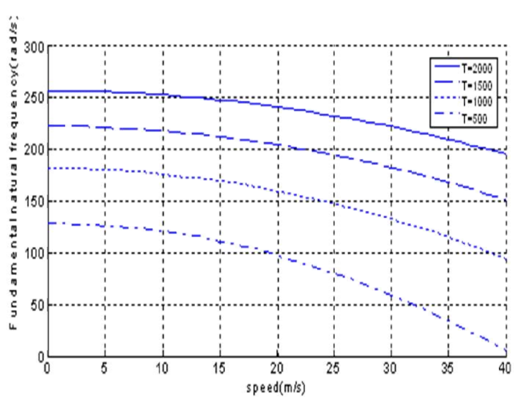


Fig.3: effect of belt speed on the first natural frequency at different initial tensions

In Fig.4, the critical speed of buckling instability is plotted against the belt tension. This figure shows that critical speeds increase with the increasing of the tension, hence, one can avoid buckling instability by increasing the initial tension of belts providing that the belt working within its allowable stress limit.

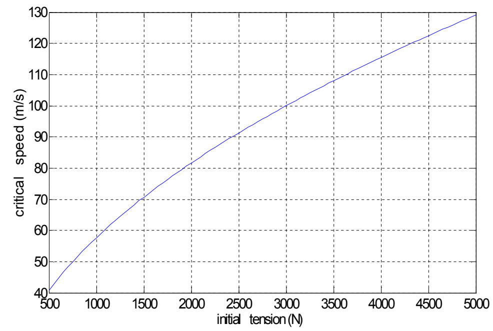


Fig.4: effect of initial tension on belt critical speeds of buckling

To check the validity of the present solution, the results of Ref.9 and the present work are shown in figure 5.The excitation frequencies are normalize relative to the natural frequency .It is important to mention that in ref.9 the Variationalmethod is used .As it is clear from the figure that, the present solution is in a good agreements with Ref.9, where the maximum error dose not exceeding 5%.

For investigating the parametric instability of fluctuated speed and the effect of belt parameters, Figs.6 to 8 are plotted In Fig.6, the boundaries of the principal instability regions at the lowest three modes are plotted for 0 to 0.5 excitation amplitude δ.

The other parameters are; mean speed=20m/s and tension=1000N . It is interesting to show that ,these regions are emerged at 340,670 and 1300 rad/s for the first ,second and third mode, respectively.

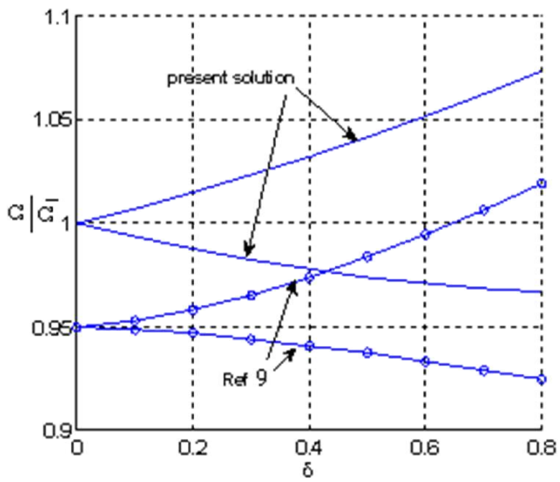


Fig.5: comparison of the present solution with Ref .9

By using Eq.15 (or referring to Fig.3 for the first mode) it is simple to show that these are the values of the twice of the natural frequencies for the corresponding modes. This is coincided well with the fundamental concept of the parametric instability of any elastic system where the primary instability regions started at 2Ω (or half time period) [4].

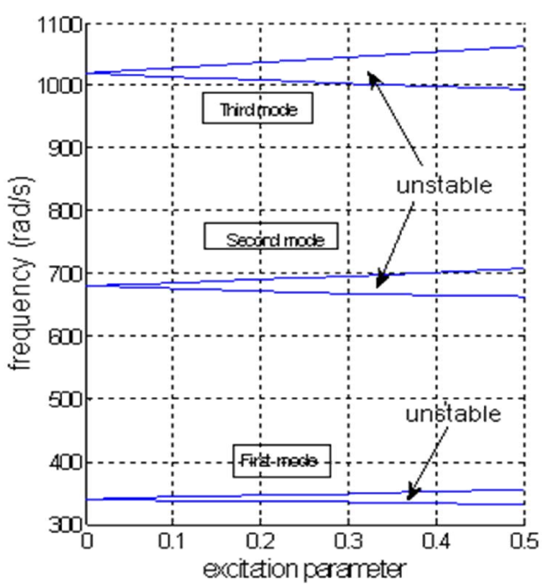
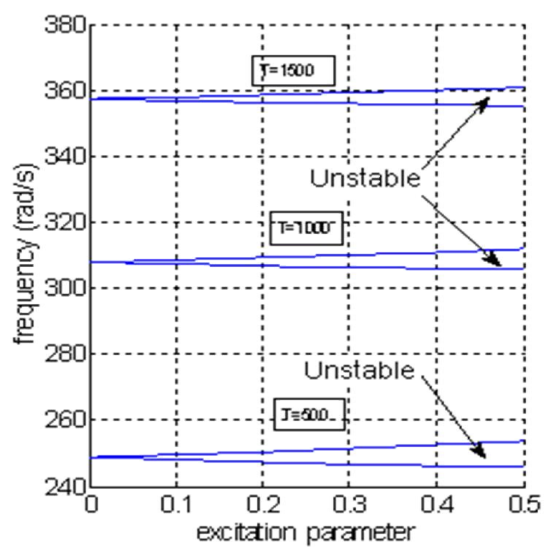


Fig.6: the lowest three modes of instability regions of belts at different excitation parameters

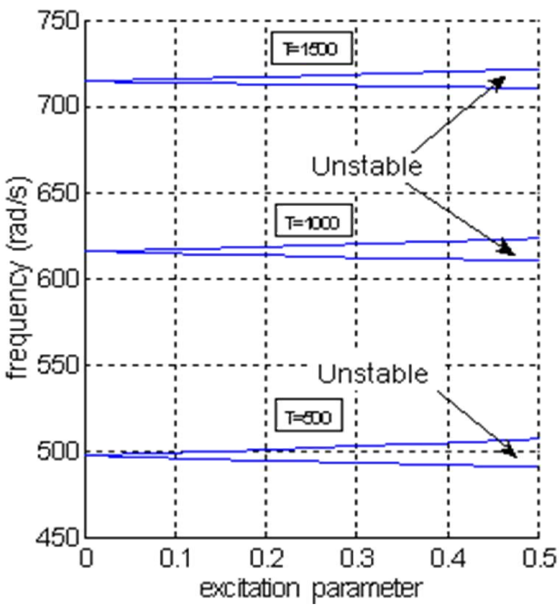
The effect of belt initial tension on the instability regions at the lowest two moes are investigated in Figs.7.It is clear from these figures that increasing the tension tend to raise these regions without affecting their sizes.

Finally, the effect of mean belt speed on the instability regions are studied in Fig.8-a and b.

In these figures the main speeds are given three different values namely (20,30 and 40 m/s).As it is clear from the figures that the speed has very significant effects on the regions of instability; firstly, it shifts the regions to a lower values (the reduction in the regions is about 15%) ;secondly, it expands the region(about six times) .This means that the speed has the major effect on the parametric instability and there is certain limits of speeds which must be not exceeded to insure safe operation.

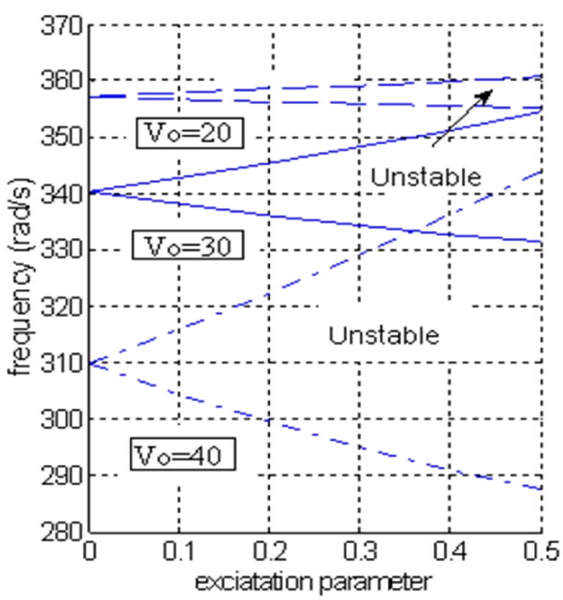


a: first mode

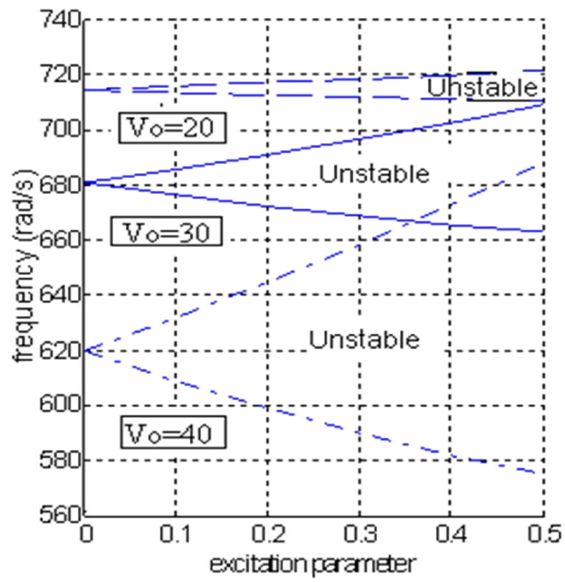


b: second mode

Fig.7: effect of initial tension on instability reigns for the first and second mode,(a) and (b)



a: first mode



b: second mode

Fig.8: effect of belt mean speed on Instability reigns for the first (a) and second mode (b)

Conclusion:

In this paper both static and parametric instabilities were investigated analytically.

The effect of the main belt parameters on stability was inspected. In case of parametric excitation caused by the fluctuated speed belt ,Bolotin method is used to separate the primary regions of instability .

From examining the results the following main conclusions can be stated;

1- The present method of solution is in a good agreement as compared with the other reported method where variation technique was employed .The results show that the error is not exceeded 5%.
2- Increasing the tension has two advantages on the stability behavior; it increases the critical speed of buckling and raises the boundaries of instabilities.

3- Increasing belt speed is limited by the critical limits where buckling instability can be initiated, as well as, it shifts instability regions to lower values with wider sizes, hence it leads to dangerous operating conditions.

References:

1. Wickert, J. A. and Mote Jr., C. D., “Classical vibration analysis of axially moving continua”, ASME Journal of Applied Mechanics 57, 1999, 738–744

2. Wickert, J. A. and Mote Jr., C. D., "Current research on the vibration and stability of axially moving materials", Shock and Vibration Digest 20(5), 1988, 3–13.

3. C. D. Mote Jr. "Parametric excitation of an axially moving string" Journal of Applied mechanics, Transactions of American Society of Mechanical Engineers 1986.35, 171–172.

4. F. Pellicano, F. Vestroni,”Complex dynamic of high-speed axially moving systems”, Journal of Sound and Vibration 258 (2006) 31–44.

5. Mockenstrum, E. M., Perkins, N. C., and Ulsoy, A. G., "Stability and limit cycles of parametrically excited,axially moving strings", ASME Journal of Vibration and Acoustics 118, 1996, 346–350.

6. Chakraborty, G., Mallik, A. K., and Hatwal, H., ‘Normal modes and near-resonance response of beams with non-linear effects’, Journal of Sound and Vibration 210(1), 1998, 19–36.

7. Pakdemirli, M., Ulsoy, A. G., and Ceranoglu, A., "Transverse vibration of an axially accelerating string",Journal of Sound and Vibration 169(2), 1994, 179–196.

8. A.S. Abrate," Vibration of belts and belt drives, Mechanism and Machine Theory" Journal of Sound and Vibration 286 ,2005, 663–668 667

9. L.Q. Chen, J.W. Zu, "Energetics and conserved functional of moving materials undergoing transverse nonlinear vibration", Journal of Vibration and Acoustics 126 ,2004, 452–455.

10. Liu ,Z and C. Huang," Evaluation of the Parametric Instability of an Axially Translating Media Using a Variational Principle", Journal of Sound and Vibration .2002. 257(5), 985–995

11. F. Pellicano, F. Vestroni, "Nonlinear dynamics and bifurcations of an axially moving beam", Journal of Vibration and Acoustics 122, 2010, 21–30.

12. G. Chakraborty, A.K. Mallik, H. Hatwal, Non-linear vibration of a travelling beam, International Journal of Non-Linear Mechanics 34 ,1999, 655–670.

13. G. Chakraborty And A. K. Mallik ," Parametrically Excited Non-Linear Traveling Beams with and without External forcing",nonlinear Dynamics 17: 301–324, 1998.

14. BolotinV.V.,"The Dynamical Stability of Elastic Systems”.San Francisco, DayIn.

عدم الاستقرار ألانبعاجي والبارامتري للسيور باستخدام طريقه بولوتن

محمود رشيد اسماعيل

جامعه النهرين - كليه الهندسه – قسم الهندسه الميكانيكية

الخلاصة:

السيور هي منظومات انتقاليه متصله وفي مثل هكذا منظومات يمكن ان يحصل عدم الاستقرار بالانبعاج والبارامتري وذلك يتوقف على كون معاملات المنظومه ثابته او متذبذبه .في هذا البحث تم حل مشكله عدم الاستقرار تحليليا .لقد تم استخدام طريقه بولوتن لايجاد الحدود التي تفصل مناطق الاستقرار عن مناطق عدم الاستقرار.تم التاكد من الطريقه الحاليه وذلك بمقارنتها مع طريقه اخرى منشوره .بينت النتائج توافقاجيدا حيث لم تتعدى نسبه الخطأ 5%.تم دراسه ناثير الشد والسرعه على الاستقرار والترددات الطبيعيه .بينت النتائج ان زياده الشد يمكن ان تحسن الاستقرار بينما يجب ان تكون السرعه او معدلها محدده ضمن قيم معينه لتجنب مخاطر عدم الاستقراربالانبعاج او البارامتري.