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Chaotic Multiple Access System Based on Orthogonal Chaotic Vector of Lorenz System

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Abstract:

Chaos phenomenon is capable of generating chaotic sequences with low cross-correlation, which can be useful for spreading the direct sequence code division multiple access (DS-CDMA) system. In this paper, a chaotic multiple access communication system based on Orthogonal Chaotic Vector (OCV) generated from Lorenz system has been proposed. The simulation results show that the performance of the proposed system is almost the same regardless of the number of users in Additive White Gaussian noise AWGN channel. The results also show that at bit-error-rate of 10^{-3}, the proposed system has achieved gains of 6 dB and 11 dB in signal-to-noise ratio over traditional DS-CDMA based on Walsh-Hadamard sequence in AWGN and Rayleigh fading channels, respectively.

Keywords: chaotic signals; Lorenz system; multiple access; DS- CDMA; orthogonal chaotic vector

I. Introduction

Recently, many works [1-6] have shown the effectiveness of applying chaotic sequences in multiple access digital communication systems to replace the quasi-orthogonal binary code sequences. For CDMA systems, sequences with low cross-correlation properties are desired. Such sequences can be produced by chaotic systems due to its sensitivity to initial conditions. A concrete proposal for the chaotic CDMA system could already be found as early as 1992 [7], followed by other similar proposals [8-11]. Mazzini et al. [12] presented the performance analysis of the chaotic CDMA system. They showed that chaos-based sequences can outperform m-sequences & Gold codes. Venkatesh and Singh [13] in 2011 suggested a scheme to improve the orthogonality of chaotic carriers used in chaotic CDMA system on a penalty of increased complexity. Zia [14] in 2014 proposed an efficient channel estimation scheme for multicarrier chaotic CDMA system based on interpolation method. However, the researchers tend to present themselves very positively in talking about chaos. Thus potential advantages of chaotic solutions are emphasized, but difficult problems are often not discussed adequately. In case of CDMA systems, the current conventional spreading codes in use are orthogonal Walsh codes. For such systems, the chaos codes cannot beat the orthogonal codes in AWGN environment [15]. Nevertheless, because of the rather bad auto-correlation properties of the orthogonal codes, there should be cases in multipath channel environment, in which chaos codes outperform orthogonal codes.

The orthogonal chaotic vectors (OCV) represent the optimal solution for the orthogonality problem of chaotic sequences since they keep having good auto-correlation properties after vectors generation. Unfortunately, there are no much publications on the performance comparison of chaotic CDMA based on OCV with traditional CDMA based on orthogonal spreading codes. Probably because it is commonly recognized that orthogonal spreading codes will generally outperform chaos map spreading codes. Furthermore, the little existing works considered the use of chaotic maps for CDMA although the chaotic flows like Lorenz are easier to be generated practically from physical electronic circuit. In this work, direct sequence DS-CDMA digital communication system based on the OCV Lorenz flow sequences (x, y, and z) has been proposed and simulated. A random time-invariant multipath channel is used for BER simulations while a RAKE receiver with enough fingers is used to capture all the energy from the multipath. Perfect channel estimation is assumed, and only downlink multiuser cases are simulated. The performance of the proposed system has been compared with traditional CDMA system.

I. Architecture of the proposed DS-CDMA based on chaotic Lorenz system

The block diagram of the proposed DS-CDMA based on Chaotic Lorenz system (CDS-CDMA) is shown in Fig.1. In the transmitter for each user (say j^{th} user) of the N_u users, the data $d^{(j)}$ is generated and modulated using BPSK modulator (QPSK or other digital modulation schemes are also applicable). The modulated data $d_m^{(j)}$ of each user are spread using a unique Lorenz chaotic sequence. The Lorenz sequences are generated by taking one of the outputs of Lorenz system (i.e. x-sequence, y-sequence, or z-sequence) and applying it to an orthogonal chaotic vector (OCV) circuitry that generates N_u Lorenz sequences orthogonal to each other for n different users. The spreading operation is simply a multiplication process between the chaotic and the BPSK modulated signals. The spread signals are added to each other and sent though the transmission channel. At the receiver, each of the N_u received signals is de-spread by multiplying it by exactly similar replica of chaotic signal at transmitter of the corresponding user. an integrate and dump operation is performed to detect the energy produced by the despread signal and finally the recovered signal is demodulated to obtain the transmitted data. The difference between the proposed system and the traditional DS-CDMA used for wireless applications is that the digital Walsh-Hadamard sequence is replaced by Lorenz OCV analogue sequence to improve the system performance and capacity.

II. Lorenz chaotic system and the orthogonal chaotic vector algorithm

The Lorenz system is a continuous-time dynamical system which belongs to the autonomous type of dynamical systems. The Lorenz system of differential equations arose from the work of meteorologist/mathematician Edward N. Lorenz, who was studying thermal variations in an air cell underneath a thunderhead [16]. The Lorenz system is three-dimensional system described by the following differential equations [17]:

$$\dot{x} = \sigma(y - x) \quad ..... (1)$$
$$\dot{y} = rx - y - xy \quad ..... (2)$$
$$\dot{z} = xy - bz \quad ..... (3)$$

Where [x y z] is the output state vector while σ, r and b are three real valued parameters. As a three-dimensional system with three parameters, the Lorenz system can lead to very complicated behavior on changing the parameter values. Choosing the standard parameter values σ = 10, r = 28, and b = 8/3 [17], the well-known "buttery" attractor is obtained, as shown in Fig.2.

III. The orthogonal chaotic vector algorithm

The non-zero values of cross-correlation between mutual chaotic sequences (x, y, and z) of Lorenz system in chaotic CDMA system result in Multiple Access Interference (MAI) between users. The effect of MAI increases as the number of users increases. In order to eliminate the effect of problem (MAI), we apply rule of Gram-Schmidt's ortho-normalization process [18] to one of the chaotic sequences (x, y, or z). The result will be a set of orthogonal chaotic vectors OCV for each chaotic sequence (x, y or z). The Gram-Schmidt ortho-normalization process for N_u orthogonal chaotic signal generators is given by [13]:

$$\hat{y}(k)^{(p)} = \frac{y(k)^{(p)} - \sum_{q=1}^{p-1}\left[\sum_{k=1}^{\beta} y(k)^{(p)}\hat{y}(k)^{(q)}\right]\hat{y}(k)^{(q)}}{\sqrt{\sum_{k=1}^{\beta}\left[y(k)^{(p)} - \sum_{q=1}^{\beta}\left[\sum_{k=1}^{\beta} y(k)^{(p)}\hat{y}(k)^{(q)}\right]\hat{y}(k)^{(q)}\right]^2}} \quad ..... (4)$$

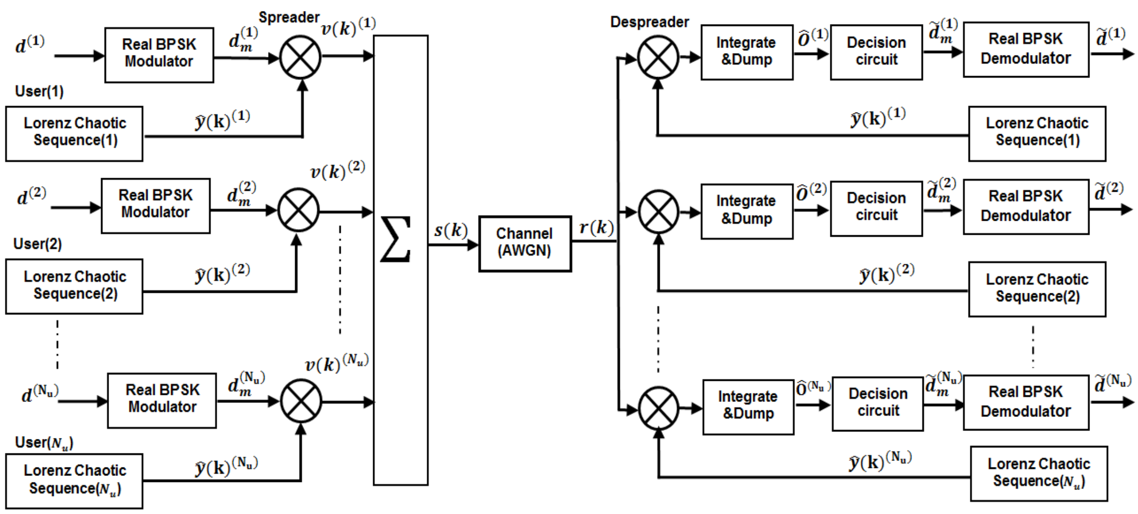

Figure 1: The proposed chaotic DS-CDMA system based on Lorenz model.

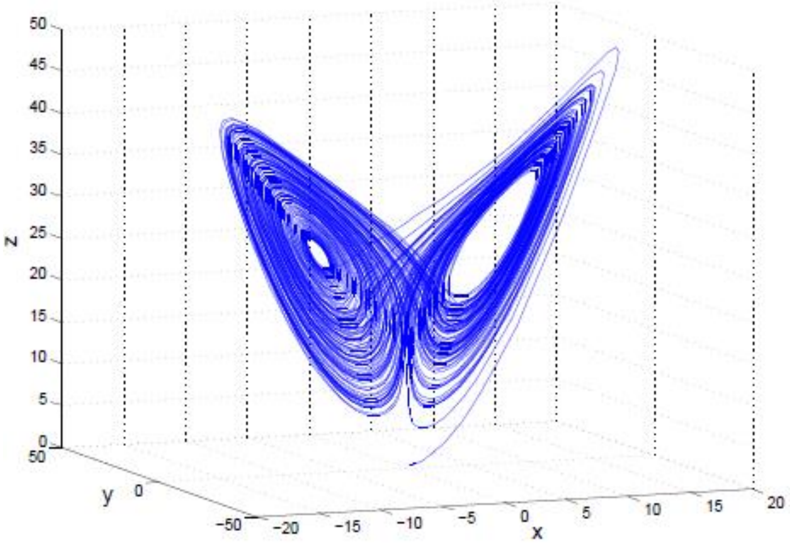

Figure 2: Lorenz attractor with parameters values: σ = 10, r = 28, and b = 8/3, initial conditions $[x_0 \; y_0 \; z_0] = [0.3 \; 0.3 \; 0.3]$.

where p = 2, 3. . . N_u. For p = 1(i.e. single user)

$$\hat{\boldsymbol{y}}(\boldsymbol{k})^{(\mathbf{1})} = \frac{\boldsymbol{y}(\boldsymbol{k})^{(\mathbf{1})}}{\sqrt{\sum_{\boldsymbol{k}=\mathbf{1}}^{\boldsymbol{\beta}}[\boldsymbol{y}(\boldsymbol{k})^{(\mathbf{1})}]^{\mathbf{2}}}} \quad \ldots.. \;(5)$$

Where $\hat{y}(k)^{(p)}$ is the chaotic carrier for p^{th} user and β is the number of chaotic samples used to transmit single binary bit (i.e. spreading factor). These OCVs are used as spreader to spread message bits and to increase the numbers of active users. The orthogonal chaotic sequences can be generated from the different outputs of Lorenz chaotic system (x, y, and z) with similar or different initial conditions. Fig.3. shows the orthogonal chaotic sequences generated using the output y of Lorenz system.

The mean value of chaotic carrier is made equal to zero, in order to avoid unwanted dc power transmission. As it was mentioned in section II, the chaotic sequences are multiplied by baseband modulated BPSK data sequence $d_m\{-1, +1\}$ to obtain the spread vector $v(k)^{(j)}$ .The transmitted signal $s(k)$ is the sum of modulated orthogonal chaotic vectors of each user and can be represented as:

$$\boldsymbol{s}(\boldsymbol{k}) = \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N_u}} \boldsymbol{v}(\boldsymbol{k})^{(\boldsymbol{j})} \quad \ldots.. \;(6)$$

IV. Performance Analysis Over AWGN Channel

Assuming that the signal is corrupted only due to AWGN, the received signal r(k) can be represented as:

$$\boldsymbol{r}(\boldsymbol{k}) = \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N_u}} \boldsymbol{V}(\boldsymbol{k})^{(\boldsymbol{i})} + \boldsymbol{\xi}(\boldsymbol{k}) \quad \ldots.. \;(7)$$

Where ξ(k) is the additive white Gaussian noise with zero mean and $N_o/2$ variance. At the receiver, it is assumed that a similar replica of spreading sequence is available and it is exactly synchronized with the transmitted one. The m^{th} decoded symbol for the j^{th} user, denoted by $\tilde{d}_m^{(j)}$, is determined using [18]:

$$\tilde{\boldsymbol{d}}_{\boldsymbol{m}}^{(\boldsymbol{j})} = \begin{cases} +\mathbf{1}, \boldsymbol{if}\; \boldsymbol{O}^{(\boldsymbol{j})} = \sum_{\boldsymbol{k}=\mathbf{1}}^{\boldsymbol{\beta}} \boldsymbol{r}(\boldsymbol{k})\hat{\boldsymbol{y}}\;(\boldsymbol{k})^{(\boldsymbol{j})} > 0 \\ -\mathbf{1}, \boldsymbol{if}\;\; \boldsymbol{O}^{(\boldsymbol{j})} = \sum_{\boldsymbol{k}=\mathbf{1}}^{\boldsymbol{\beta}} \boldsymbol{r}(\boldsymbol{k})\hat{\boldsymbol{y}}\;(\boldsymbol{k})^{(\boldsymbol{j})} \le \mathbf{0} \end{cases} \quad \ldots.. \;(\mathbf{8})$$

Without the loss of generality, we consider the probability of error for the first symbol. Omitting the subscripts of the variables $\tilde{d}_m^{(j)}$ and $O^{(j)}$ for the sake of brevity, the decision parameter of the j^{th} user is given by:

$$O^{(j)} = d^{(j)} \sum_{k=1}^{\beta} \left[\hat{y}(k)^{(j)}\right]^2 + d^{(j)} \sum_{i=1, i\neq j}^{N_u} \sum_{k=1}^{\beta} \left(\hat{y}(k)^{(i)} \hat{y}(k)^{(j)}\right)^2 + \sum_{k=1}^{\beta} \xi(k)\hat{y}(k)^{(j)} \quad \ldots\ldots (9)$$

Since chaotic vectors used for each user is ortho-normal to each other, the second term in equation (9) causing MAI will be equal to zero. Assuming, that $O^{(j)}$ has a Gaussian distribution, the BER for j^{th} user can be written as [13]:

$$BER^{(j)} = \frac{1}{2} erfc\left(\frac{E\left(O^{(j)}|d^{(j)} = +1\right)}{\sqrt{\left(2\, var\left(O^{(j)}|d^{(j)} = +1\right)\right)}}\right) \quad \ldots\ldots (10)$$

Where the mean value of $(O^{(j)}|d^{(j)} = +1)$ is given by:

$$E\left(O^{(j)}|d^{(j)} = +1\right) = \beta E\left[\left(\hat{y}(k)^{(j)}\right)^2\right] = E_b \quad (11)$$

where E_b is energy per bit. The variance is given by:

$$Var\left(O^{(j)}|d^{(j)} = +1\right) = var\left[\sum_{k=1}^{\beta} \left[\hat{y}(k)^{(j)}\right]^2\right] + \beta \frac{N_O}{2} E\left[\left(\hat{y}(k)^{(j)}\right)^2\right] = \frac{E_b N_O}{2} \quad (12)$$

Substituting equations (11) & (12) in equation (10), we get:

$$BER^{(j)} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_O}}\right) \quad (13)$$

From equation (13) it can be concluded that BER performance of the proposed system is independent on the number of users and spread factor which is a unique and strong feature not exist in other multiple access systems.

V. Performance Analysis Over Rayleigh Fading Channel

Assuming that the channel is a slow Rayleigh fading channel, let α is a Rayleigh distributed random variable denoting fading gain. Then it can be shown that the BER of jth user in symbol duration is:

$$BER_{\alpha}^{(j)} = \frac{1}{2} erfc(\gamma) \quad \ldots\ldots (14)$$

where

$$\gamma = \frac{\alpha E_b}{N_o} \quad \ldots\ldots (15)$$

Since α is Rayleigh-distributed random variable, γ (the received instantaneous signal to noise ratio per bit) will be chi-square distributed and has the form [18],

$$f_{Rayleigh} = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}, \quad \gamma \geq 0 \quad \ldots\ldots (16)$$

Where

$$\bar{\gamma} = E[\gamma] = E[\alpha]. \frac{E_b}{N_o} \quad \ldots\ldots (17)$$

Therefore, the average BER for jth user is:

$$BER_{Rayleigh} = \int_0^{\infty} BER_{\alpha}(\gamma)\, f_{Rayleigh}(\gamma)\, d\gamma, \quad \ldots\ldots (18)$$

The last equation shows that in case of Rayleigh fading channel, there is no clear relation whether the BER performance of the proposed system is independent on the number of users and spread factor or not. Therefore, the simulation results may show us more clear view about this issue.

VI. Simulation Results

A complete simulation model for the proposed OCV Lorenz based CDS-CDMA system shown in Fig.1 has been implemented using MATLAB package 7.13.0.564 (R2011b). From other hand, another simulation model for traditional CDMA system based on orthogonal Walsh-Hadamard code with the same simulation parameters has also been implemented for the purpose of performance comparison. In all simulation results the spreading factor is chosen to be 64 in both systems. The parameters of the generated Lorenz sequence were σ = 10, r = 28, and b = 8/3 while the initial conditions were $[x_0 \quad y_0 \quad z_0]$ = [0.155 0.15 0.155]. The number of users has changed from single user to a maximum of 16 users.

- **Simulation results in AWGN channel:**

Fig.4 shows the BER performances of CDMA based on OCVs generated from individual outputs of Lorenz system (x, y, and z) for single user transmission in AWGN channel together with traditional CDMA system. It can be noticed from this figure that the performances of OCVs generated using different outputs of Lorenz system are not the same. The OCV associated with output y has the best performance. Therefore, it will be selected for all the remaining simulation results. Also it is obvious in this figure that OCVs outperforms the traditional CDMA. For instant, at BER=10^{-3} a gain of 6 dB in E_b/N_0 can be obtained when we use OCV (y-output) as compared with the traditional CDMA. Fig.5 depicts the BER performances when the number of users is 4. Once again, the OCV based CDMA outperforms the traditional CDMA. The improvement starts to occur when SNR more than -5 dB. At BER=10^{-3}, a gain of 6 dB in E_b/N_0 is obtained. The improvement can be attributed to good combination of auto and cross-correlation properties of Lorenz OCVs. In other words, OCV sequences have ideal auto-correlation properties and acceptable cross-correlation ones Walsh sequences have ideal cross-correlation properties but worse auto-correlation ones. Fig.6 shows the performance of the proposed OCV of Lorenz based CDMA with different number of users. It is obvious from this figure that the performance is almost the same regardless of the number of users. This result confirms the conclusion mentioned early in section V upon the derivation of equation (13).

- **Simulation results in Rayleigh fading channel:**

Fig.7 shows the BER performances of CDMA based on OCVs generated from individual outputs of Lorenz system (x, y, and z) for single user transmission in 3-tap random time-invariant Rayleigh multipath channel [14] together with traditional CDMA system. It can be seen in this figure, that all OCVs generated from different Lorenz system outputs outperform the traditional CDMA. However, the best performance results are obtained in the case of (output y). It is worth noted that the improvement in the case Rayleigh fading channel is more than AWGN case. For instant, at BER=10^{-3}, a gain of 11 dB in SNR is obtained Rayleigh fading channel while it was 6 dB in AWGN channel.

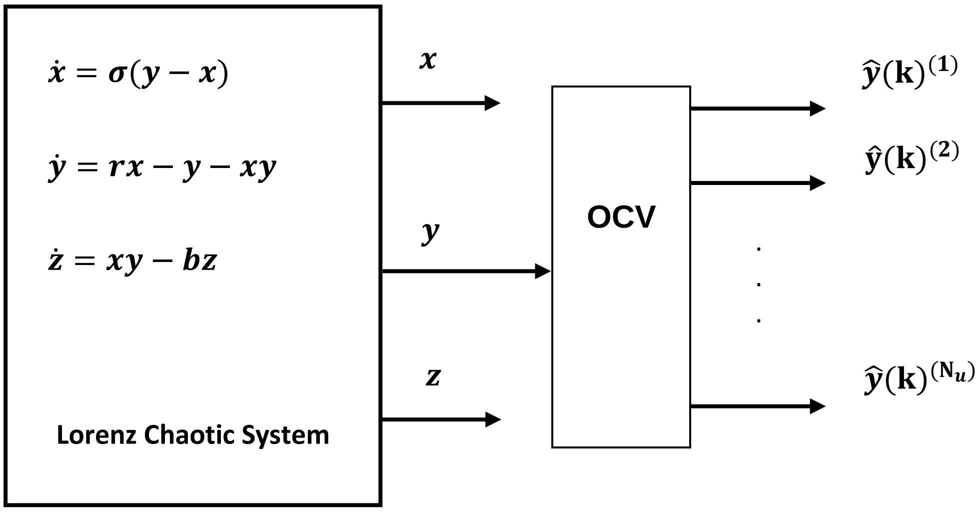

Figure 3: Generation of orthogonal chaotic sequences from Lorenz system.

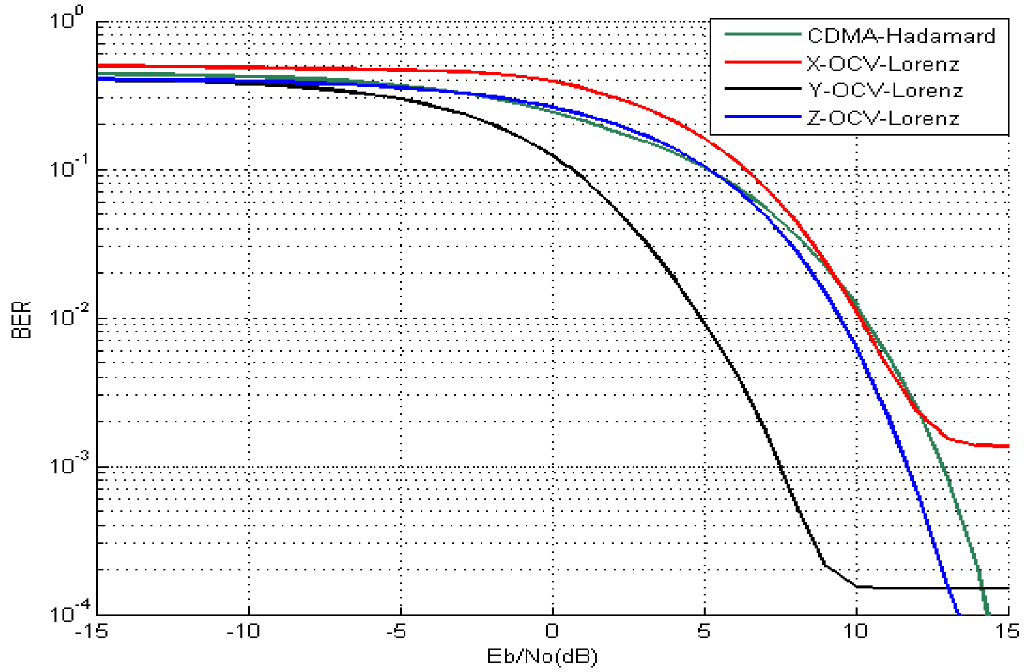

Figure 4: Performance of OCV of Lorenz chaotic sequences x, y and z, and the orthogonal Walsh-Hadamard sequence for single user transmission in AWGN channel.

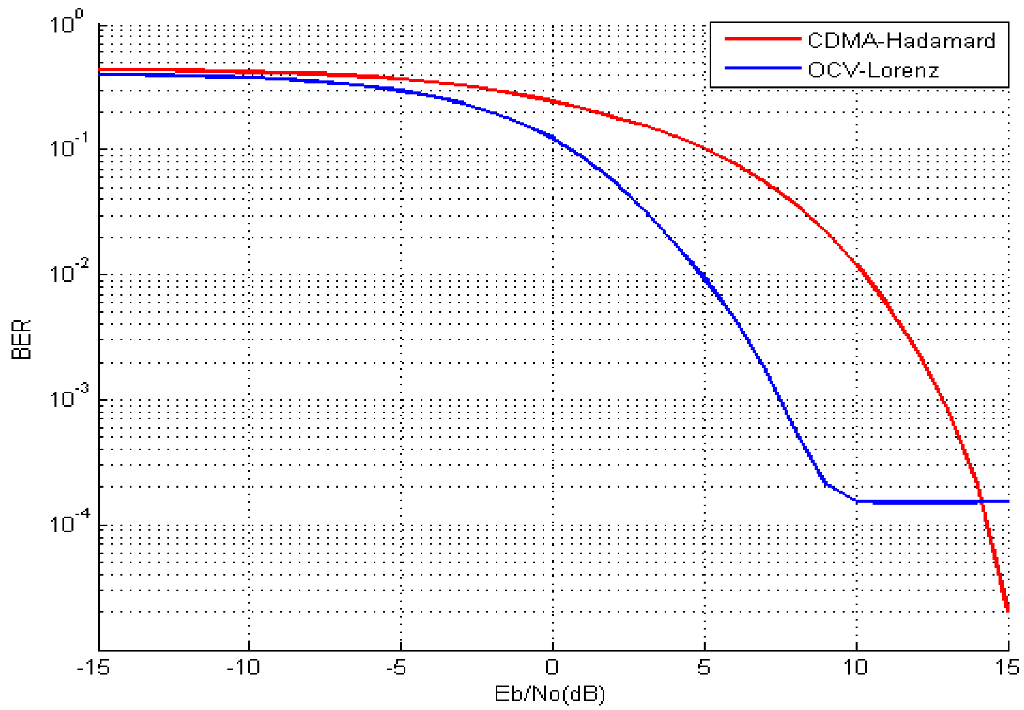

Figure 5: Performance of OCV of Lorenz based CDMA (sequence y) and traditional CDMA for 4 users' transmission in AWGN channel

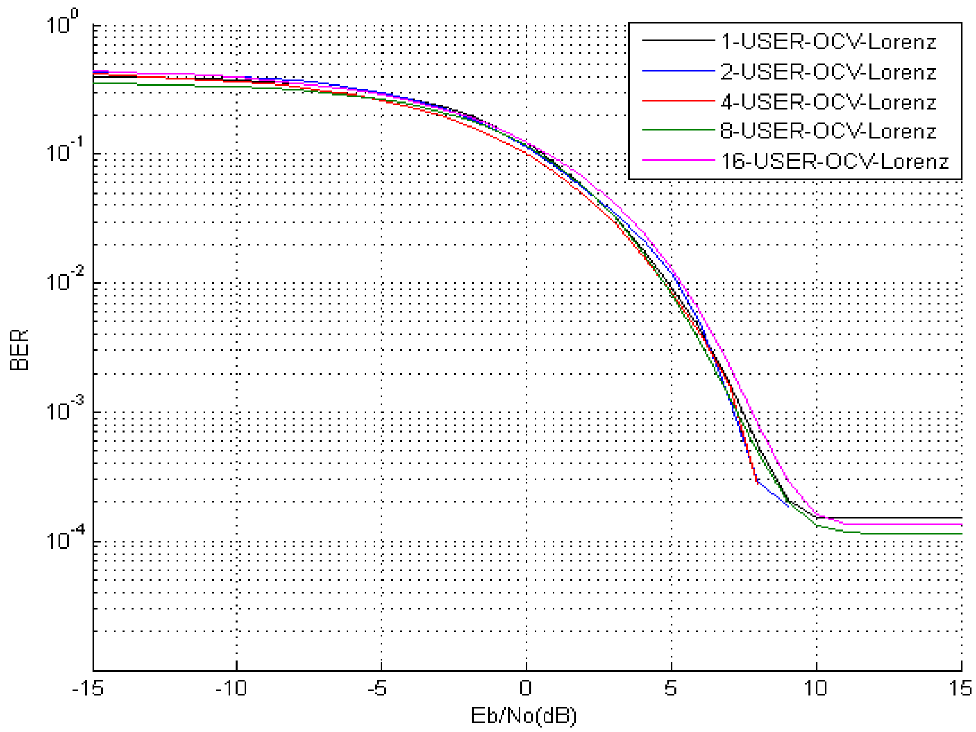

Figure 6: Performance of OCV of Lorenz based CDMA for different number of users in AWGN channel.

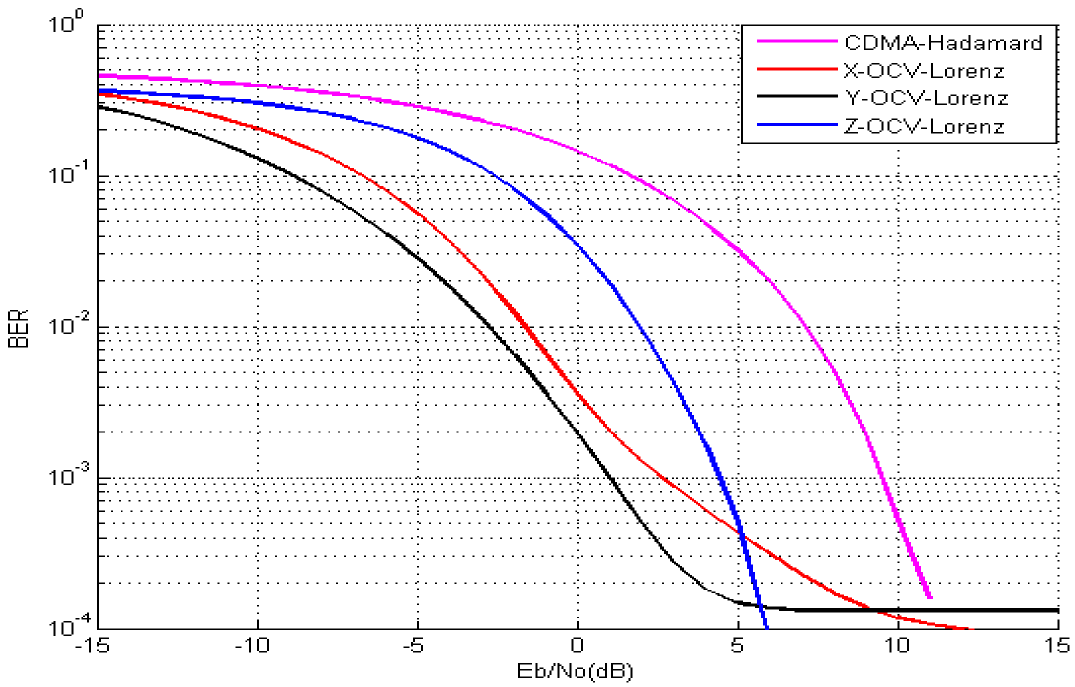

Figure 7: Performance of OCV of Lorenz chaotic sequences x, y and z, and the orthogonal Walsh-Hadamard sequence for single user transmission in Rayleigh fading channel.

Fig.8 shows the BER performance comparison when the number of users is 4. Here, it can be easily noticed that the OCV Lorenz based CDMA outperforms the traditional CDMA. Once again, the improvement in the case Rayleigh fading channel is more than AWGN case. For instant, at BER=10^{-3}, a gain of 8 dB in SNR is obtained in Rayleigh fading channel while it was 6 dB in AWGN channel. Finally, Fig.9 depicts the performance of the proposed OCV of Lorenz based CDMA with different number of users. It is obvious from this figure that the performance is improved as the number of users is decreased which is the similar case in traditional CDMA systems. However, this was not the case in AWGN channel discussed in Fig.6 where the performance was the same regardless the number of users. But in all cases the performance of the proposed scheme is better than the traditional CDMA for the similar number of users as we have seen for an example case N_u=4 in the previous figure.

VII. Conclusions

The performance of CDMA system can be improved by the use of orthogonal chaotic vector generated from Lorenz chaotic system using Gram-Schmidt ortho-normalization process as a spreading sequence. The orthogonal chaotic vector has ideal auto-correlation properties and acceptable cross-correlation ones. This good combination is the reason behind its good performance in both AWGN and fading channels as compared with the traditional Walsh-Hadamard sequence which has ideal cross-correlation properties but worse auto-correlation ones. The orthogonal chaotic vectors generated from different Lorenz system outputs have different performances; so, the one of the best performance should be selected to obtain the best improvement. Finally, as the theoretical analysis and simulation results show, the orthogonal chaotic vectors have almost the same performance regardless of the number of users or the spreading factor in AWGN which is a unique feature as compared with other multiple access techniques.

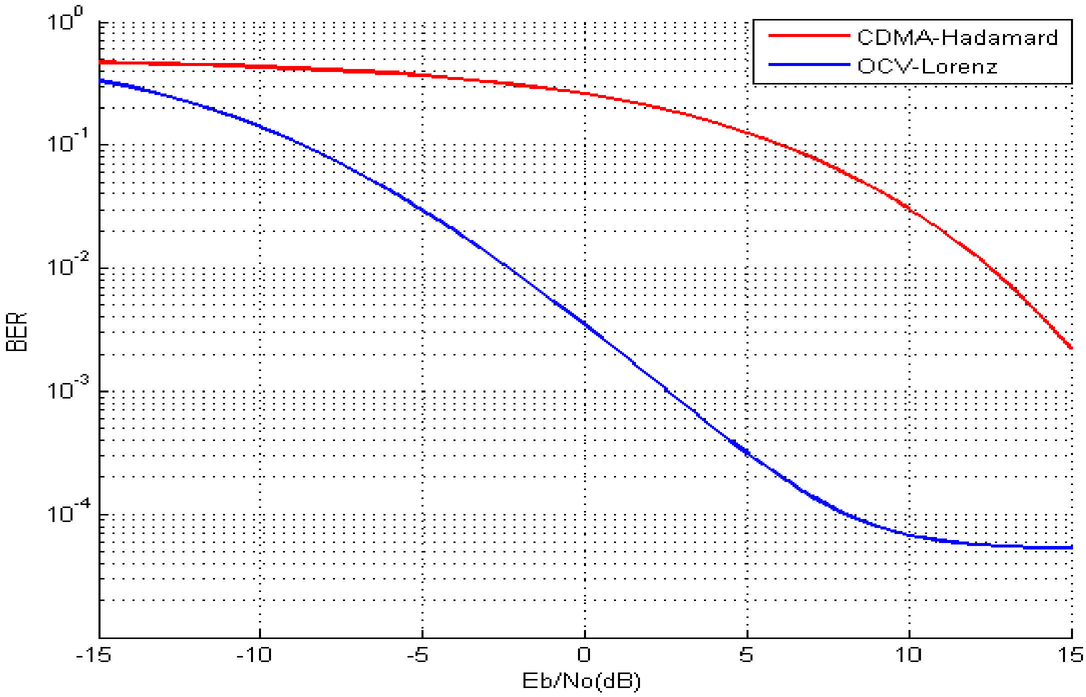

Figure 8: Performance of OCV of Lorenz based CDMA (sequence y) and traditional CDMA for 4 users' transmission in Rayleigh fading channel.

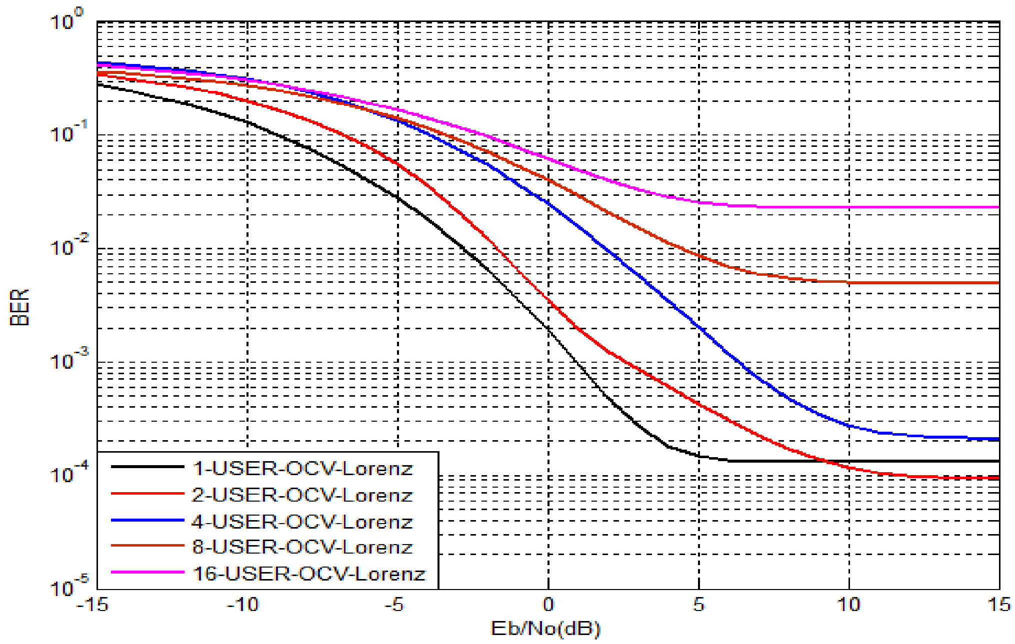

Figure 9: Performance of OCV of Lorenz based CDMA for different number of users in Rayleigh fading channel.

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نظام الوصول المتعدد الفوضوي المستند على متجه فوضوي متعامد نوع لورينز

حكمت نجم عبدالله **علي عبد راضي**

الجامعة المستنصرية/ كلية الهندسة / قسم الهندسة الكهربائية

الخلاصة:

بالامكان استثمار ظاهرة الفوضى لتوليد سلاسل فوضوية تمتلك دوال ارتباط قليلة المقدار والتي تكون مفيدة لنشر طيف اشارة نظام السلسلة المباشرة ذات الوصول المتعدد (DS-CDMA). لقد تم في هذا البحث اقتراح نظام اتصالات وصول متعدد مستند على متجه فوضوي متعامد متولد من نظام لورينز(Lorenz) ، ولقد بينت نتائج المحاكاة بأن اداء النظام المقترح في قناة كاوس الضوضائية يبقى ثابتا بغض النظر عن عدد المستخدمين. كذلك فقد بينت النتائج ان النظام المقترح يمكن ان يحقق تحسينات في نسبة الأشارة الى الضوضاء مقاديرها 6 ديسيبل و 11 ديسيبل في قناتي كاوس الضوضائية ورايلي التوهينية على التوالي عند نسبة خطأ مقدارها 10^{-3} مقارنة بنظام السلسلة المباشرة ذات الوصول المتعدد التقليدي والمستند على سلسلة والش-هادمارد (Walsh-Hadamard).