Optimum Setting of PID Controller using Particle Swarm Optimization for a Position Control System

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Abstract:
The goal of this paper is to present a study of tuning the Proportional–Integral–Derivative (PID) controller for control the position of a DC motor by using the Particle Swarm Optimization (PSO) technique as well as the Ziegler & Nichols (ZN) technique. The conventional Ziegler & Nichols (ZN) method for tuning the PID controller gives a big overshoot and large settling time, so for this reason a modern control approach such as particle swarm optimization (PSO) is used to overcome this disadvantage. In this work, a third order system is considered to be the model of a DC motor. Four types of performance indices are used when using the particle swarm optimization technique. These indices are ISE, IAE, ITAE and ITSE. Also study the effect of each one of these performance indices by obtaining the percentage overshoot and settling time when a unit step input is applied to a DC motor. A comparison is made between the two methods for tuning the parameters of PID controller for control the position of a DC motor. The first one is tuning the controller by using the Particle Swarm Optimization technique where the second is tuning by using the Ziegler & Nichols method. The proposed PID parameters adjustment by the Particle Swarm Optimization technique showed better results than the Ziegler & Nichols’ method. The obtained simulation results showed good validity of the proposed method. MATLAB programming and Simulink were adopted in this work.

Keywords: Position of a DC motor, PID controller, Particle Swarm Optimization, Ziegler and Nichols’ method, and performance indices.

1. Introduction
The DC motor is a power actuator device that transforms electrical energy to mechanical energy. Due to its simplicity and continuously control characteristics, DC motors have been widely used in many industrial applications such as robotic manipulators and electric cranes. The DC motors provide a suitable control for the position and speed deceleration or acceleration. For these reasons, the researchers have paid high attention to position and speed control of DC motor and prepared several methods to control its position and speed. One of these methods is a Proportional–Integral–Derivative (PID) controller which is usually used for controlling the speed and position. Tuning of the parameters of PID controller is very important because they have great effect on the control system performance and stability. There are various methods for tuning the PID controller parameters, such as Ziegler and Nichols (Z-N) method [1], Particle Swarm Optimization (PSO) [2, 3, 4, 5, 6 and 7], Genetic Algorithm (GA) [8, 9, 10 and 11] and many other methods such as in [12, 13, 14 and 15]. Ziegler–Nichols method is a conventional well known method; but sometimes, doesn’t give a good tuning; yielding a big overshoot. For this reason several methods have been proposed to overcome this PID tuning drawback; such as using the natural selection and search method based on PSO algorithm [10]. In the current work we have focused on making a comparison between the Z-N and PSO methods for tuning the PID controller. MATLAB programming and Simulink were utilized. The simulation results obtained, have demonstrated good dynamic behavior of the PSO based PID controller. The results revealed a perfect position tracking with minimum overshoot, minimum steady state error, less rise and settling time and realization of better performance in comparison with the conventional PID controller that use the Ziegler and Nichols (Z-N) method.

2. Description of DC Motor Model
DC motors are most suitable for adjusting and control of wide range positions and speeds. The DC motor speed is proportional to its applied voltage, while its torque is proportional to its current. The speed control of a DC motor depends on many factors such as variable battery tapings, variable supply voltage and the control of resistors. Figure (1) shows a model for simple motor. In this figure, the armature resistance ($R_a$) is connected in series with an inductance ($L_a$), whereas the voltage ($V_a$) represents the back emf voltage (back electromotive force) induced in the armature during the rotation [1]. The physical parameters values are given in table 1, [12].
The rotor and the shaft are assumed to be rigid. The armature voltage $V$ in Volts is driven by a voltage source and considered as the input. The measured variables are the angular velocity of the shaft $\omega$ in radians per second, the shaft angle $\theta$ in radians, and the motor torque, $T$, in \( \text{mN} \).

The torque is related to the current of armature, $i_a$, in Ampere by a constant factor $K$:

$$ T = k i_a $$

The back electromotive force (emf), $V_b$, is related to the $\omega$ by the relation expressed as:

$$ V_b = K \omega = K \frac{d\theta}{dt} $$

Based on the combined Newton’s and Kirchhoff’s law, we can write the following equations from figure (1):

$$ J \frac{d^{2}\theta}{dt^2} + b \frac{d\theta}{dt} = K i_a \tag{3} $$

$$ L_a \frac{di}{dt} + R_a i_a = V - K \frac{d\theta}{dt} \tag{4} $$

If the Laplace transform is used, the equations (3) and (4) can be described as:

$$ J s^2 \Theta(s) + b s \Theta(s) = K I_a(s) \tag{5} $$

$$ L_a s I_a(s) + R_a I_a(s) = V(s) - K s \Theta(s) \tag{6} $$

$I_a(s)$ in equation (6) can be re-write as:

$$ I_a(s) = \frac{V(s) - K s \Theta(s)}{R_a + L_a s} \tag{7} $$

By substituting (7) in (5) we can obtain

$$ J s^2 \Theta(s) + b s \Theta(s) = \frac{K}{R_a + L_a s} \left( V(s) - K s \Theta(s) \right) \tag{8} $$

Equation (8) can be described as:

$$ J s \omega(s) + b \omega(s) = \frac{K}{R_a + L_a s} \left( V(s) - K \omega(s) \right) \tag{9} $$

Figure (2) illustrates the block diagram of the above equations for the DC motor.

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$$ J s^2 \Theta(s) + b s \Theta(s) = \frac{K}{R_a + L_a s} \left( V(s) - K s \Theta(s) \right) \tag{8} $$

Equation (8) can be described as:

$$ J s \omega(s) + b \omega(s) = \frac{K}{R_a + L_a s} \left( V(s) - K \omega(s) \right) \tag{9} $$

Figure (2) illustrates the block diagram of DC-Motor Model

From equations (8) and (9) the following transfer functions can be obtained:

$$ \frac{\omega(s)}{V(s)} = \frac{K}{(R_a + L_a s)(J s + b) + K^2} \tag{10} $$

$$ \frac{\theta(s)}{V(s)} = \frac{K}{s[(R_a + L_a s)(J s + b) + K^2]} \tag{11} $$

With the above parameters values given in table (1), the transfer function in equation (11) will be as:

$$ G(s) = \frac{\theta(s)}{V(s)} = \frac{0.01}{(0.005 s^2 + 0.06 s + 1) * \frac{1}{s}} $$

### 3. Problem Description

The PID controller consists of Proportional, Integral and Derivative gains. This controller can be described as:

$$ C(s) = K_p + \frac{K_i}{s} + K_d s \tag{12} $$

Where $K_p$, $K_i$, $K_d$ represent respectively the proportional gain, integral gain, and derivative gain. For best system performance these parameters must be tuned. The feedback control system for DC Motor with PID controller is shown in Figure (3). The signals in this figure can
be expressed as: the reference input, \( r(t) \), the error signal, \( e(t) \), and the output variable, \( y(t) \). In this Figure, \( G(s) \) represent the transfer function of the DC Motor, where the \( C(s) \) represent the PID controller, given in Equation 12.

**Figure (3):** The feedback control system for DC Motor.

The most important advantage of using the PID controllers is its ability to eliminate the steady-state error of the step input response because of the integral action effect. The plant used in this work is a DC motor and its model described in Eq. 11.

Furthermore, performance index is used here to compute the optimal values of the parameters of the designed PID controller in order to achieve the required specification. There are four types of the performance indices that can be used when tuning process for a PID-controller is required. These four indices can be described as

\[
ISE = \int_0^\infty e^2(t)dt \tag{13}
\]

\[
IAE = \int_0^\infty |e(t)|dt \tag{14}
\]

\[
ITAE = \int_0^\infty t|e(t)|dt \tag{15}
\]

\[
ITSE = \int_0^\infty t e^2(t)dt \tag{16}
\]

Therefore, for tuning a PID controller; based on the PSO-technique, the above Performance indexes will be used as the objective function in order to find the optimum set of the PID controller’s gains which make the DC Motor system having a minimum performance index.

**4. Ziegler and Nichols’ (Z-N) method**

Ziegler and Nichols have two methods for tuning PID controllers. The first is for the open-loop tuning, whereas the second which is used in this work is based on the step response of the close-loop system. In close-loop method the ultimate period and critical gain need to be found. The critical gain \( (K_u) \) is determined by adjusting the controller gain \( (K_u) \) carefully until the system response reaches the case of sustained oscillations. At this time, the ultimate period \( (P_u) \) must be determined. The values of the PID controller can then be determined as it is described in table 2.

**Table (2):** The Ziegler and Nichols’ method for close loop system.

<table>
<thead>
<tr>
<th>Controller</th>
<th>( K_p )</th>
<th>( T_i )</th>
<th>( T_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>( 0.5k_u )</td>
<td>_</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>( 0.45k_u )</td>
<td>( 1.2k_p / p_u )</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>( 0.6k_u )</td>
<td>( 2k_p / p_u )</td>
<td>( k_pop_u / 8 )</td>
</tr>
</tbody>
</table>

**5. Particle Swarm Optimization (PSO)**

The PSO can be defined as an optimization technique based on evolutionary computation. The first PSO was firstly developed in 1995 [10], and in 1998, the original PSO was improved after adding the inertia weight in the modified PSO. During iteration, this inertia weight decreases linearly [5]. This type of PSO is commonly used by researcher and is adopted in this work.

**6. Tuning of PID controller based on PSO Technique**

In PSO, a potential solution to the problem is represented by a particle. The flying of each particle is adjusted with respect to the experience of its own flying and the experience of the flying of its companion. The particle is represented by a point in the D-dimensional space. Where \( XI = (x1, x2, \ldots, xD) \) represents the position of the \( i^{th} \) particle. Each particle gives minimum fitness value is recorded and is considered as the best particle position and defined as \( PI = (p1, p2, \ldots, pD) \). The particle is subjected as pbest. The gbest represents the best particle in the pbest population that having minimum fitness with respect to others particles in this population. The \( i^{th} \) particle velocity is represented by \( VI = (v1, v2, \ldots, vD) \). Equations (17) and (18) that is shown bellow will be used for updating the Velocity and position of the particles.

\[
v_{id}^{n+1} = w \cdot v_{id}^n + c_1 \cdot rand() \cdot (P_{id}^n - x_{id}^n)
+ c_2 \cdot rand() \cdot (P_{gd}^n - x_{id}^n) \tag{17}
\]

\[
x_{id}^{n+1} = x_{id}^n + v_{id}^{n+1} \tag{18}
\]

In equation (17) above, both \( c_1 \) and \( c_2 \) are constants \((> 0)\)
and each is equal to 1.494 [5], rand() represents the random function and its values is between 0 and 1, \( w \) is the inertia weight which have a value
between 0 and 1, and in this work its value is considered as 0.729 as was suggested in [5]. Finally the iteration is denoted by n.

By using equation (17) above, the new velocity of the particle’s $v_{id}^{n+1}$ can be calculated. Equation (18) is then used to calculate the new position. The performance index is used to measure the fitness of each particle. This fitness is concerned with the problem that we want to solve. To make a balance between the global and local search capability, the inertia weight, $w$, is inserted in equation (17) and is chosen for convergence to be $w=0.729$ as mentioned above. This value of $w$ has been adopted in this work.

6.1. PID Controller Tuning; Based on PSO Implementation

PSO technique is used off-line in order to tune the PID gain ($K_p$, $K_i$, and $K_d$) using the model in Eq.11. Firstly the PSO algorithm produces a matrix of initial particles in search space. Every initial particle can be assumed as a candidate solution for the PID gains. The values of these controllers gain are assumed to be from 0 to 50. In this work, the swarm size (number of particles) is assumed to be 30. A (3 x Swarm size) matrix is used to obtain each of the position and the velocity in PSO algorithm by using equations (17) and (18). The PSO technique, explained above, is used to obtain optimal values of PID controller gains and achieving good response. These optimal values of PID controller gains are resulted from minimizing the performance index; expressed in Eqs.13-16 above.

7. The Simulations Result

In the conventional method that uses the Z-N for tuning the PID controller, the response of the plant gives high overshoot, while a good response with small overshoot and better performance was obtained when a PID controller was tuned by PSO algorithm. Different results are obtained for using different performance indices in tuning the PID controllers by a PSO technique as shown in Tables 3 and 4. Table 4 gives the results for using both methods mentioned above for tuning the PID controller. These results are obtained from evaluating the unit step response performance such as overshoot and settling time. The corresponding plot for the unit step response for both methods is shown in Fig. 4 to Fig. 8.

<table>
<thead>
<tr>
<th>Tuning Method</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-N PID</td>
<td>73.2</td>
<td>97.600</td>
<td>13.725</td>
</tr>
<tr>
<td>PSO-PID (ISE)</td>
<td>50.0000</td>
<td>0.0668</td>
<td>50.0000</td>
</tr>
<tr>
<td>PSO-PID (IAE)</td>
<td>30.0000</td>
<td>0.0000</td>
<td>32.4471</td>
</tr>
<tr>
<td>PSO-PID (ITSE)</td>
<td>50.0000</td>
<td>0.0988</td>
<td>50.0000</td>
</tr>
<tr>
<td>PSO-PID (ITAE)</td>
<td>50.0000</td>
<td>36.8297</td>
<td>15.5195</td>
</tr>
</tbody>
</table>

Table 3: The optimized parameters of PID controller

<table>
<thead>
<tr>
<th>Tuning Method</th>
<th>Over-Shoot (%)</th>
<th>Settling Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-N PID</td>
<td>68</td>
<td>50</td>
</tr>
<tr>
<td>PSO-PID (ISE)</td>
<td>5</td>
<td>38</td>
</tr>
<tr>
<td>PSO-PID (IAE)</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>PSO-PID (ITSE)</td>
<td>5</td>
<td>38</td>
</tr>
<tr>
<td>PSO-PID (ITAE)</td>
<td>28</td>
<td>27</td>
</tr>
</tbody>
</table>

Table 4: The Performance of a Step Response for PID Controllers with different performance index

Figure 4: Motor Position vs. Time using Integral Square Error (ISE)
Figure 5: Motor Position vs. Time using Integral Absolute Error (IAE)

Figure 6: Motor Position vs. Time, using Integral Time Square Error (ITSE)

Figure 7: Motor Position vs. Time using Integral Time Absolute Error (ITAE)

Figure 8: Motor Position vs. Time, using Ziegler and Nichols method

8. Discussion and Conclusion
From the obtained results in table 4, we can notice that the PID controllers based on conventional method (Z-N) have high overshoot and long settling time compared to that of the proposed method when using PID controllers based on PSO technique with different performance indices. The four performance indices that were used with the PSO technique gives a good and satisfied time response when a unit step time response is considered to be the input to a DC motor. The results show the advantage of the proposed PID controllers that use the PSO optimization technique with different performance indices. Besides, the results indicate an ability of the modern optimization technique, such as the PSO, to improve the performance and properties of the time response when a PID controller is used by suggest the optimal values for the $K_p$, $K_i$ and $K_d$ which are the parameters of the PID controller. Furthermore, the designed PID controllers based on PSO technique with different performance indices have closely the same performances and properties except the case of using the ISE and ITSE as a performance index. Where in the case of using ISE and ITSE a long settling time is appeared in the response. Finally, we can notice from table 4, that the case of using the IAE as a performance index is the best one. The proposed method expressed above could be also used for controlling higher order systems as well as nonlinear systems.

9. References
أختيار القيم المثلى لسيطرة مثالية لمن نوع التناسبي-التكاملي-التفاعلية باستخدام تقنية ضرب الطر秘书 للسيطرة على الزاوية الدورانية لمحرك تيار مستمر

أحمد خلف حمودي
قسم هندسة السيطرة والنظم – الجامعة التكنولوجية

الخلاصة:

الهدف من هذا البحث هو تقديم دراسة عن كيفية اختيار القيم المثلى لجيدة للسيطرة على من نوع التناسبي-التكاملي-التفاعلية للسيطرة على الزاوية الدورانية إخراج مثالي باستخدام تقنية ضرب الطر秘书 بطرق مختلفة. أن الطرق التقليدية المستخدمة سابقاً كطريقاً زكلي ونييكلس تعطي محرك تيار مستمر. هذه القيم العليا تتبع أيضاً في هذا السياق يتم استخدام تقنية ضرب الطر秘书 لتحديد القيم المثلى. ويتم استخدام أربعة أنواع من مؤشر القيم المثلى في هذه الدراسة. وهي اعتماد دائم للأخيار المثلى لمحرك تيار مستمر. يتم اختبار طرق مختلفة لقياس القيم المثلى من الطرق التقليدية، وذلك يعتمد على القيمة المثلى للتحكم اليدوي. تظهر النتائج أن طريقة الضرب الطر秘书 تقدم أربعة أنواع من مؤشر القيم المثلى. كما أظهرت النتائج المستخلصة أن تقنية الضرب الطر秘书 هي أفضل من طريقة زكلي ونييكلس.